

Non-linear clustering observables for models beyond Λ CDM

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Based on work by: Renate Mauland-Hus, Dennis Fremstad, Ruan Cheng-Zong,
Bartolemao Fiorini, Guilherme Brando, ...

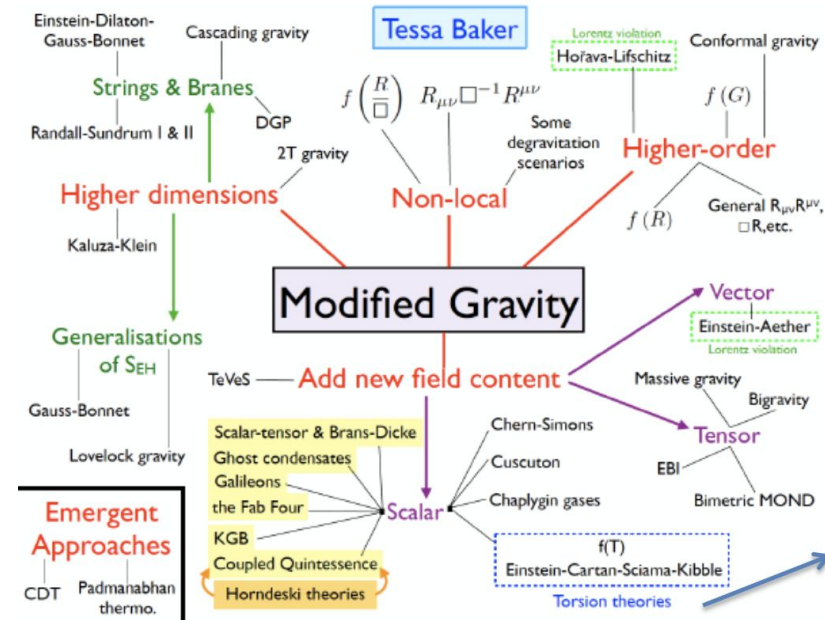
Motivation

We want to constrain models beyond Λ CDM (modified gravity, dark energy models, dark matter models, etc) with current and upcoming data from surveys.

This often requires simulations to produce mock galaxy catalogues or to produce theoretical predictions (e.g. non-linear matter power-spectrum).

Such simulations of models beyond Λ CDM has a long history now spanning ~ 20 years (for modified gravity).

I will review a bit how we do such simulations, what has been done in the past, and show how anyone can compute theoretical predictions (i.e. emulators for their favourite model and observable).



How do we simulate models beyond LCDM?

Modified Gravity

Typical models that have been studied and simulated in the literature are on the form of GR with an additional scalar degree of freedom couples to the matter sector (giving us a fifth-force). Often have a screening mechanism (implying a very non-linear PDE) to recover GR in high density regions

$$\ddot{\mathbf{x}} = -\nabla\Phi$$

$$\nabla^2\Phi = 4\pi G\delta\rho$$

N-body equations of motion GR

$$\ddot{\mathbf{x}} = -\nabla\Phi - \nabla\phi$$

$$\nabla^2\Phi = 4\pi G\delta\rho$$

$$\square\phi = f(\phi, \rho)$$

N-body equations of motion MG

How do we simulate models beyond Λ CDM?

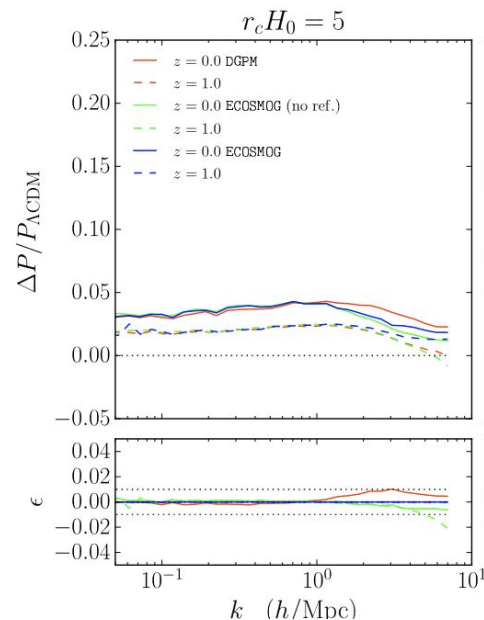
Modified Gravity

Simulations of such models goes back to the mid 2000s.
Main challenge for most models is that the PDE is non-linear and have bad convergence properties.

Standard method: relaxation with multigrid acceleration
= make initial guess and iteratively correct it with Newtons method till convergence is reached. Often done in the quasi-static limit (no time-evolution for perturbations giving us a Poisson-like equation).

Code comparison project 10 years ago (1506.06384) shows that we know well how to do these models.

Not hard, just takes more time: typically 2-5 times the speed of a normal N-body (but nothing compared to the cost of baryonic simulations).



Results from different codes showing good agreement

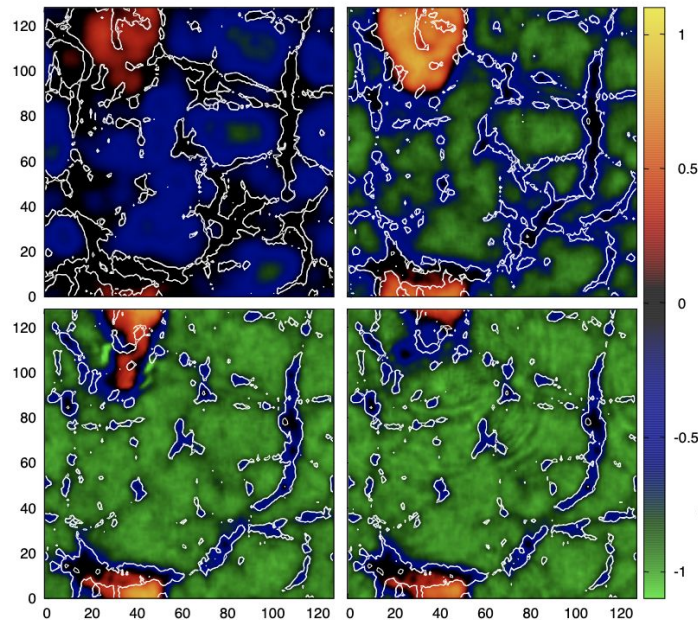
How do we simulate models beyond LCDM?

Modified Gravity

People have also gone beyond the quasi-static limit to study effects like scalar radiation, domain walls, spontaneous scalarization, etc.

Even more expensive. For most models studied, the results for the clustering are practically the same as in the quasi-static limit.

Most simulations done for a small section of models (e.g. $f(R)$, DGP), but thought to be “representative”.



Domain walls in non quasi-static sims (1302.1774)

How do we simulate models beyond LCDM?

Dark Matter

Axion models (fuzzy dark matter)

$$\square \phi + m^2 \phi = 0$$

Simple scalar field. But extremely rapid oscillations makes it challenging to simulate. Can be factored out using WKB to yield a Schrödinger equation:

$$i\hbar \psi = -\frac{\nabla^2 \psi}{2m} + m\Phi \psi \quad \rho = |\psi|^2, \quad v = \frac{\nabla(\psi/|\psi|)}{2m}$$

Can be solved using standard methods for the Schrödinger equation: operator splitting, spectral methods or solved using SPH.

For the mass-range of most interest ($\sim 10^{-23}$ eV) this is still very expensive and often limits the boxsize to the order of $O(1)$ Mpc/h.

Simulations are crucial to get predictions for what happens on small scales, but not feasible for large volumes.

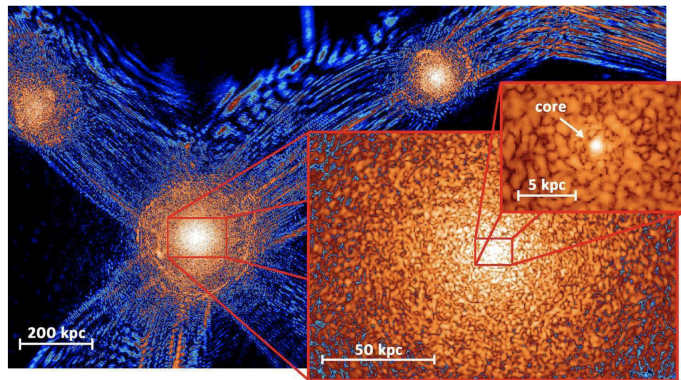


Figure 2: A slice of density field of ψ DM simulation on various scales at $z = 0.1$. This scaled sequence (each of thickness 60 pc) shows how quantum interference patterns can be clearly seen everywhere from the large-scale filaments, tangential fringes near the virial boundaries, to the granular structure inside the haloes. Distinct solitonic cores with radius $\sim 0.3 - 1.6$ kpc are found within each collapsed halo. The density shown here spans over nine orders of magnitude, from 10^{-1} to 10^8 (normalized to the cosmic mean density). The color map scales logarithmically, with cyan corresponding to density $\lesssim 10$.

Schive, Chiueh, Broadhurst

Emulating the matter power-spectrum

For using the data from weak lensing surveys, we require down to non-linear scales.

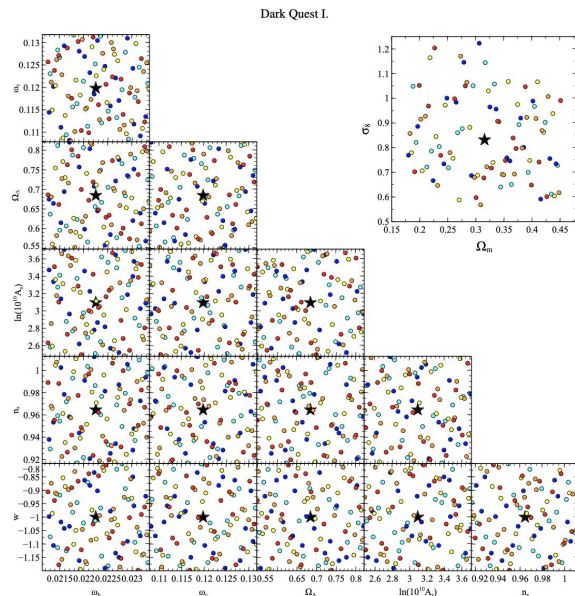
$$P(k, z | \Omega_m, h, A_s, n_s, \dots)$$

One can do this semi-analytically (e.g. HMCode, ReACT, ...) or full simulation based.

This latter approach consists of sampling the parameter-space, running simulations and then using machine learning tools to create an emulator (= interpolator).

This has been done by many groups for Λ CDM leading to emulators like EuclidEmulator2, FrankenEmu, CosmicEmu, DarkQuest, NGenHaloFit, BACCO, ...

Some of these took millions of CPU hours to create. If you want to do this for your favourite model beyond Λ CDM then you have to do it from scratch. Expensive. Should we use this much computing power on such models? Can we do this more cheaply?



Fast approximate methods

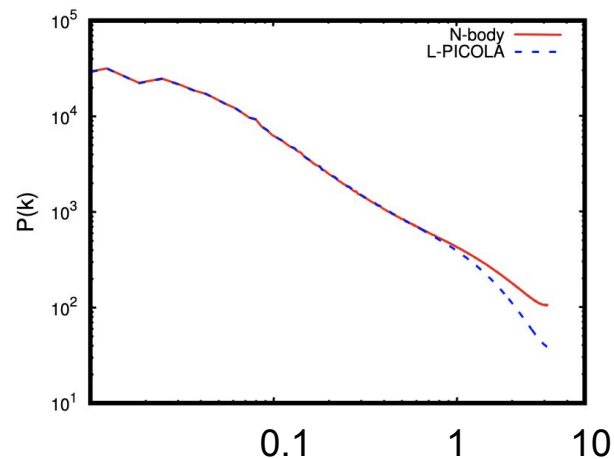
N-body simulations are expensive because we have to resolve orbits of dark matter particles in halos. Requires high mass and force-resolution and small time-steps => costly with a high memory footprint.

Faster to use simple PM simulations: use a fixed mesh and compute forces using Fourier transforms. Upshot is “cheap” force evaluations and downside is low force-resolution (if the mesh is not too large).

This can be combined with the COLA method: solve the EOM in a frame that follows the path predicted by LPT. By construction correct particle trajectories on large scales no matter how large the time-steps.

This allows us to do simulations very cheaply (using few timesteps) at the cost of giving up on some small scale accuracy. Tuning the simulations parameters, one can easily get 1% up to $k \sim 1 \text{ h/Mpc}$, but much better than this is hard.

Exists methods of “correcting” the outputs (e.g. Potential Gradient Decent 1804.00671) on small scales if needed.



Comparison of full N-body with fast approximate methods for matter power-spectrum.

Useful tool for this: FML library

C++ library for working in parallel with particles and grids and solving PDEs (<https://github.com/HAWinther/FML/>).

Contains PM N-body code for LCDM, DE and MG models (long list) with massive neutrinos and on-the-fly analysis.

For MG models it contains methods for adding in screening (1403.6492).

Fairly easy to add another model. E.g. 2209.01666 implemented general Hordenski models Hi-COLA (<https://github.com/Hi-COLACode/Hi-COLA>).

Theoretical predictions for weak lensing surveys

Goal: We want to use simulations to create an emulator for the

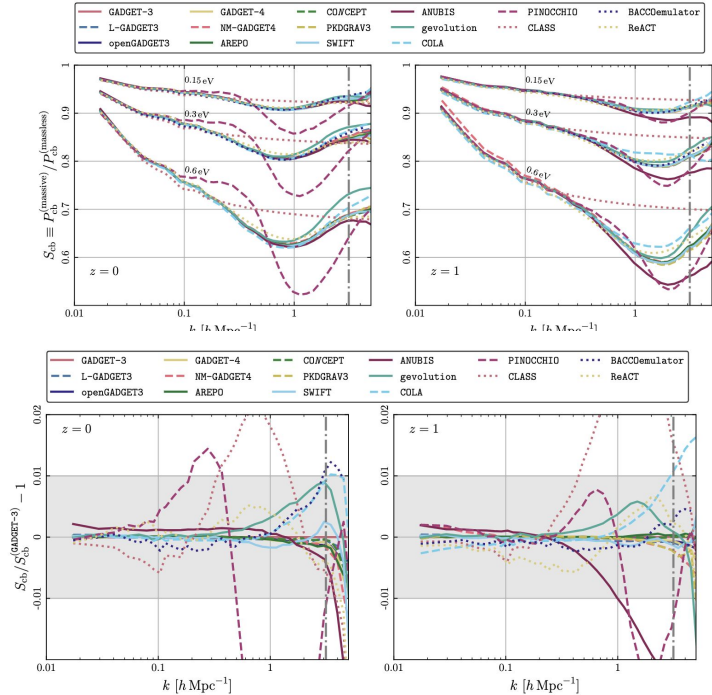
$$r = P(k, z | \Omega_m, h, A_s, n_s, \dots, \text{MG params}) / P_{\Lambda\text{CDM}}(k, z | \Omega_m, h, A_s, n_s, \dots)$$

Approach: simulate two models: 1) the beyond LCDM model 2) LCDM. Get the ratio of the two for a given set of parameters.

Advantage: a lot of the error we do on small scales “factors out”. A lot of the cosmological parameter dependence factors out. Curves are often smooth which makes it easier to emulate.

Then combine this with a LCDM emulator to get what you want. Builds on all the great work already done.

Errors can (should) be included in the covariance when fitting to data.



Example: LCDM+nu relative to LCDM.

Ratio percent level accurate up to $k \sim 3$. From 2211.12457

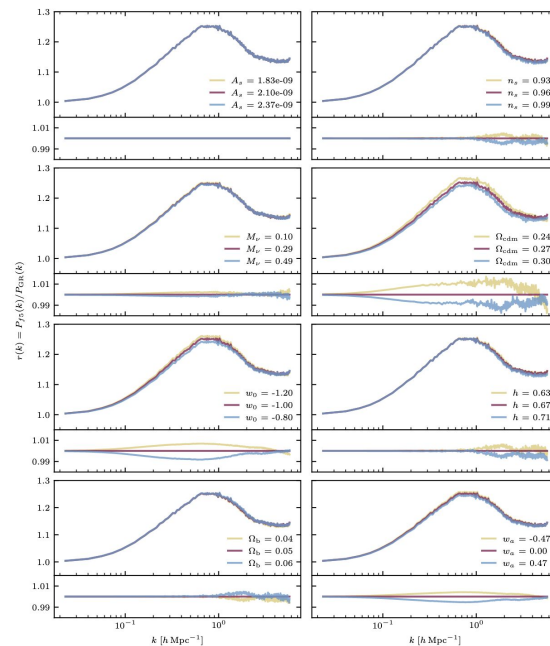
Theoretical predictions for weak lensing surveys: Modified gravity models

Take the simple $f(R)$ model as an example.

Just one extra free parameter fR_0 .

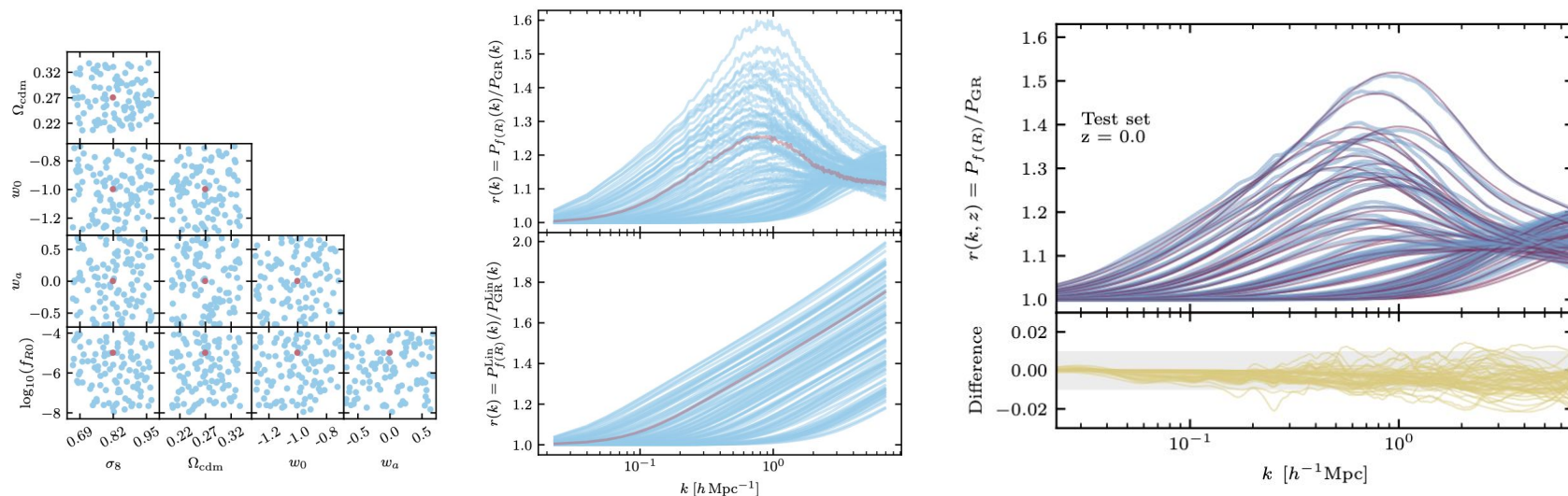
Boost found to (practically) depend only on Ω_m and σ_8

Approximation: using screening method of (1403.6492) instead of solving full non-linear field equation. Accurate to the percent level.



Cosmological parameter dependence on boost for fixed σ_8 showing its fairly insensitive to other parameters

Theoretical predictions for weak lensing surveys: Modified gravity models



One simulation here = 10min on one node. Total simulation time = 1 day using 4 nodes.

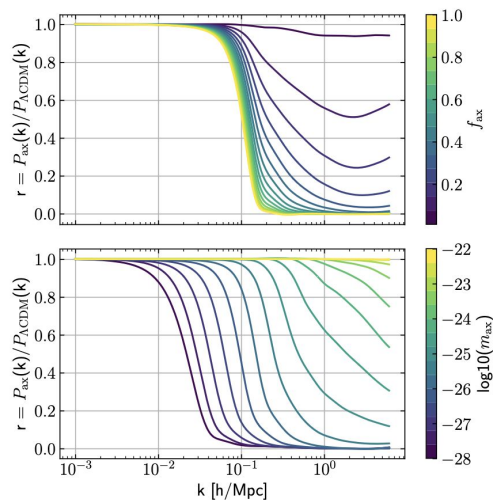
Theoretical predictions for weak lensing surveys: Mixed axion models

Models where DM is a mix of CDM and axions.

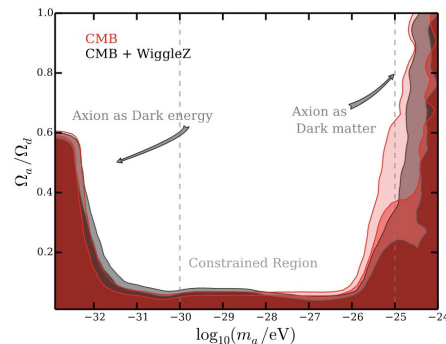
Two free parameters: axion fraction and axion mass.

Very expensive to simulate.

Approximation: effect of axions only in initial conditions. Seems to be a good approximation atleast when compared to SPH simulations (1801.08144).



Results from final emulator



Constraints on such models
on linear scales (1410.2896)

One simulation here = 5min on one node. Total simulation time = 1 day using 2 nodes.

Summary

Emulating clustering statistics in models beyond LCDM does not have to be very expensive if one uses fast approximate methods.

We can emulate the boost of our observables relative to LCDM and then build on what has already been done for LCDM.

The accuracy can be made “good enough” and in any case errors can (should) be included as a theoretical error when using it to do fits.

You do not have to be an expert on simulations to do this.

We have made (and will release) a pipeline that one can simply download, implement your model of choice and run without the need of supercomputers to produce an emulator for your favourite model. Almost anyone has the computational resources to do this.

The same approach can also be used to generate mock galaxy catalogues. Shown to give good results for a wide variety of observables (power-spectrum, bispectrum, voids, ... see e.g. 2208.01345).