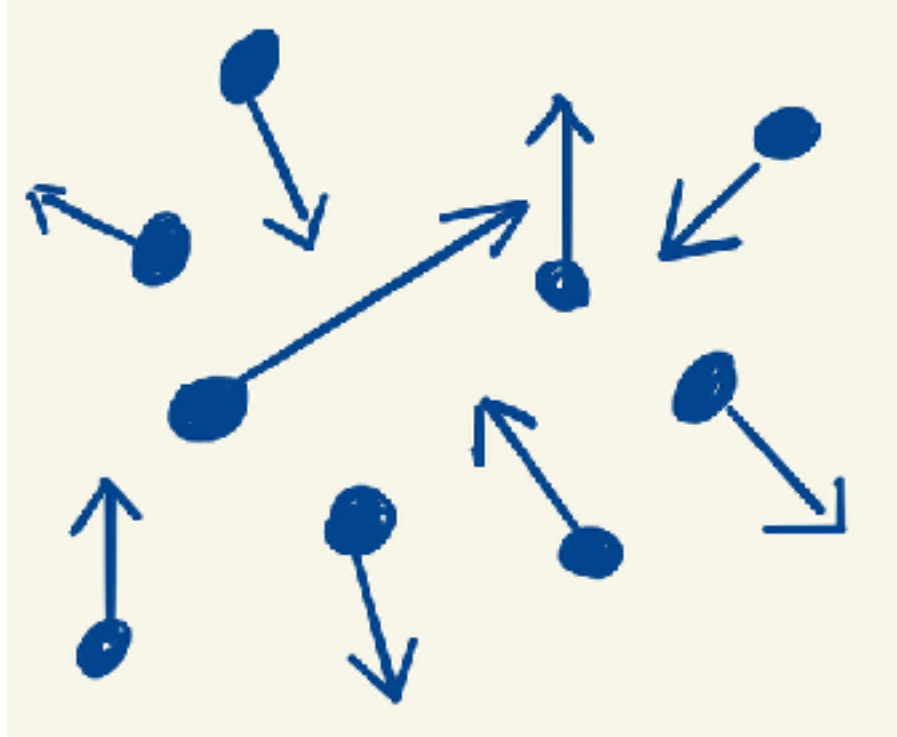


On the  
Effective Field Theory of  
Large Scale Structure

# What is a fluid?



wikipedia: credit  
National Oceanic and Atmospheric  
Administration/  
Department of Commerce

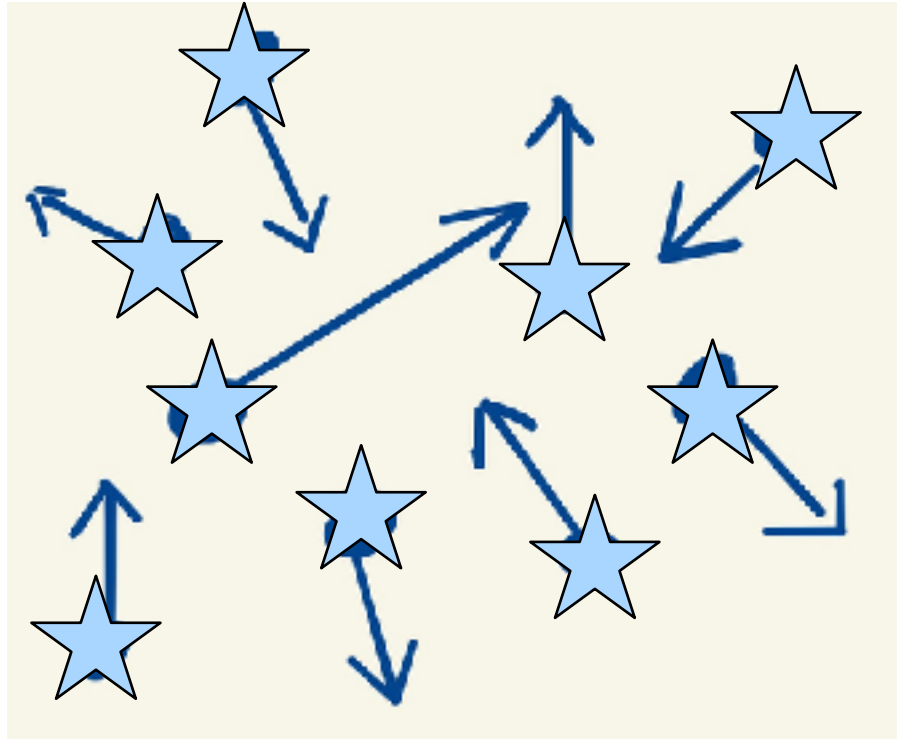


$$\partial_t \rho_\ell + \partial_i (\rho_\ell v_\ell^i) = 0$$

$$\partial_t v_\ell^i + v_\ell^j \partial_j v_\ell^i + \frac{1}{\rho_\ell} \partial_i p_\ell = \text{viscous terms}$$

- From short to long
- The resulting equations are simpler
- Description arbitrarily accurate
  - construction can be made without knowing the nature of the particles.
- short distance physics appears as a non trivial stress tensor for the long-distance fluid

# Do the same for matter in our Universe



credit NASA

with Baumann, Nicolis and Zaldarriaga **JCAP 2012**

with Carrasco and Hertzberg **JHEP 2012**

- From short to long
- The resulting equations are simpler
- Description arbitrarily accurate

– construction can be made without knowing the nature of the particles.

- short distance physics appears as a non trivial stress tensor for the long-distance fluid

$$\tau_{ij} \sim \delta_{ij} \rho_{\text{short}} \left( v_{\text{short}}^2 + \Phi_{\text{short}} \right)$$

# Dealing with the Effective Stress Tensor

- For long distances: expectation value over short modes (integrate them out)

$$\langle \tau_{ij}(\vec{x}, t) \rangle_{\text{long fixed}} = f_{\text{very complicated}} \left( \{H, \Omega_m, \dots, m_{\text{dm}}, \dots, \rho_\ell(x)\}_{\text{past light cone}} \right)$$

At *long* wavelengths  $\Downarrow$  Taylor Expansion

$$\langle \tau_{ij}(\vec{x}, t) \rangle_{\text{long fixed}} = \int^t dt' \left[ c(t, t') \frac{\delta \rho_\ell}{\rho}(\vec{x}_\text{fl}, t') + \mathcal{O}((\delta \rho_\ell / \rho)^2) \right]$$

- Equations with only long-modes

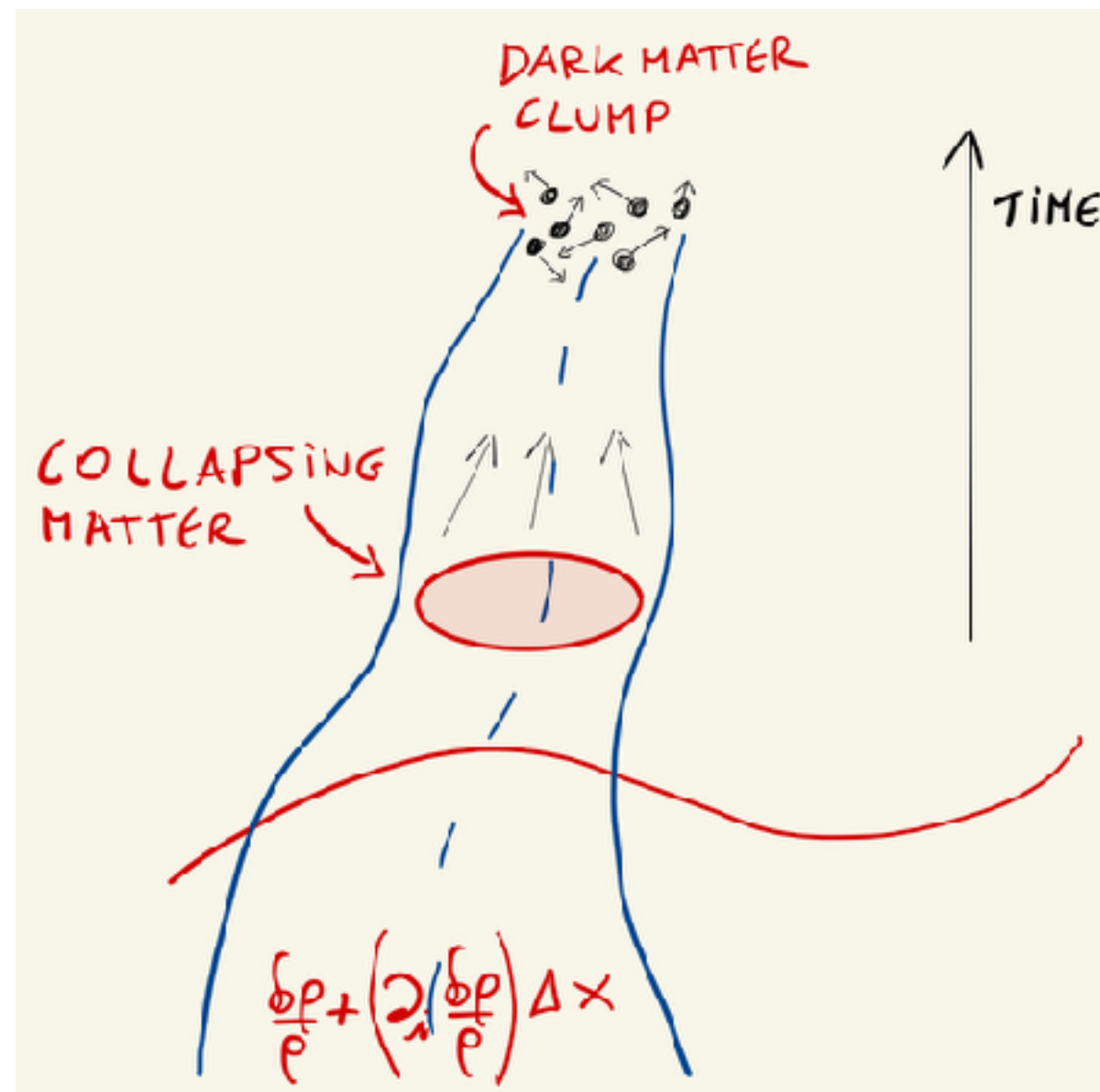
$$\partial_t v_\ell^i + v_\ell^j \partial_j v_\ell^i + \partial_i \Phi_\ell = \partial_j \tau^{ij}$$

$$\tau_{ij} \sim \delta \rho_\ell / \rho + \dots$$

every term allowed by symmetries

- each term contributes as factor of

$$\frac{\delta \rho_\ell}{\rho} \sim \frac{k}{k_{\text{NL}}} \ll 1$$





# Perturbation Theory within the EFT

- In the EFT we can solve iteratively  $\delta_\ell, v_\ell, \Phi_\ell \ll 1$ , where  $\delta_\ell = \frac{\delta\rho_\ell}{\rho}$

$$\nabla^2 \Phi_\ell = H^2 (\delta\rho_\ell / \rho)$$

$$\partial_t \rho_\ell + H \rho_\ell + \partial_i (\rho_\ell v_\ell^i) = 0$$

$$\partial_t v_\ell^i + v_\ell^j \partial_j v_\ell^i + \partial_i \Phi_\ell = \partial_j \tau^{ij}$$

$$\tau_{ij} \sim \delta\rho_\ell / \rho + \dots$$


- Two scales:

$$k \text{ [Mean Free Path Scale]} \sim k \left[ \left( \frac{\delta\rho}{\rho} \right) \sim 1 \right] \sim k_{\text{NL}}$$

# Perturbation Theory within the EFT

- Solve iteratively some non-linear eq.  $\delta_\ell = \delta_\ell^{(1)} + \delta_\ell^{(2)} + \dots \ll 1$

- Second order:

$$\partial^2 \delta_\ell^{(2)} = \left( \delta_\ell^{(1)} \right)^2 \Rightarrow \delta_\ell^{(2)}(x) = \int d^4 x' \text{ Greens}(x, x') \left( \delta_\ell^{(1)}(x') \right)^2$$

- Compute observable:

$$\langle \delta_\ell(x_1) \delta_\ell(x_2) \rangle \supset \langle \delta_\ell^{(2)}(x_1) \delta_\ell^{(2)}(x_2) \rangle \sim \int d^4 x'_1 d^4 x'_2 (\text{Green's})^2 \langle \delta_\ell^{(1)}(x'_1)^2 \delta_\ell^{(1)}(x'_2)^2 \rangle$$

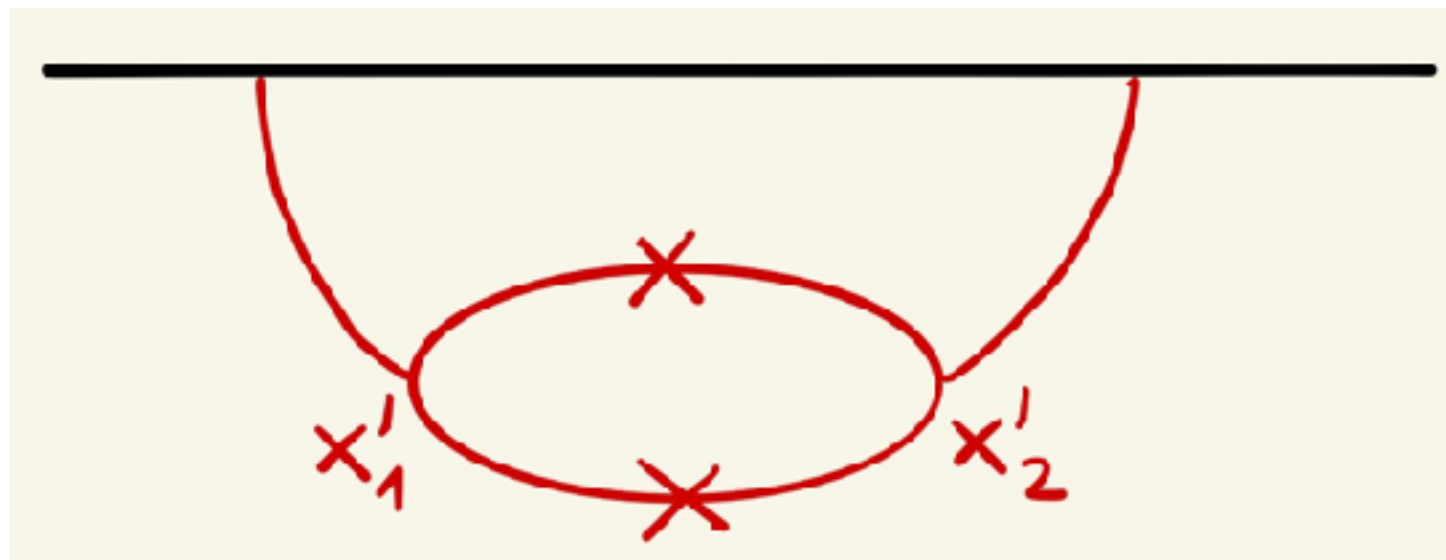
- We obtain Feynman diagrams

- Sensitive to short distance

$$x'_2 \rightarrow x'_1$$

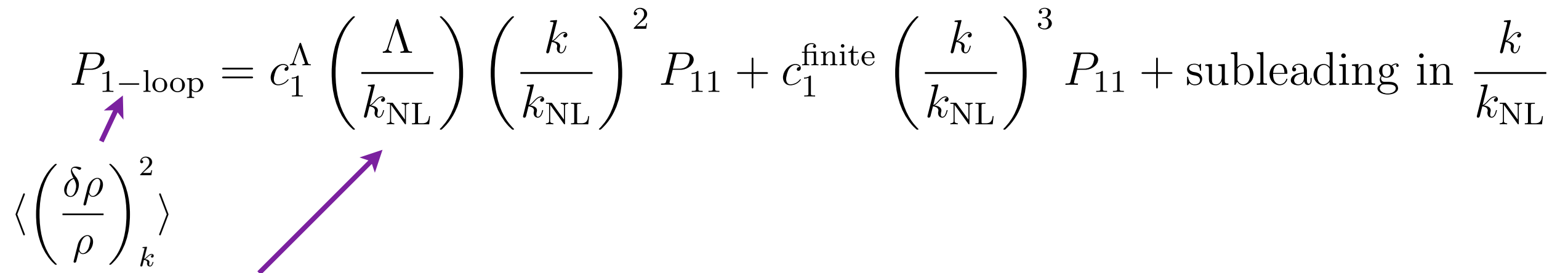
- Need to add counterterms from  $\tau_{ij} \supset c_s^2 \delta_\ell$  to correct

- Loops and renormalization applied to galaxies



- Regularization and renormalization of loops (no-scale universe)  $P_{11}(k) = \frac{1}{k_{\text{NL}}^3} \left( \frac{k}{k_{\text{NL}}} \right)^n$

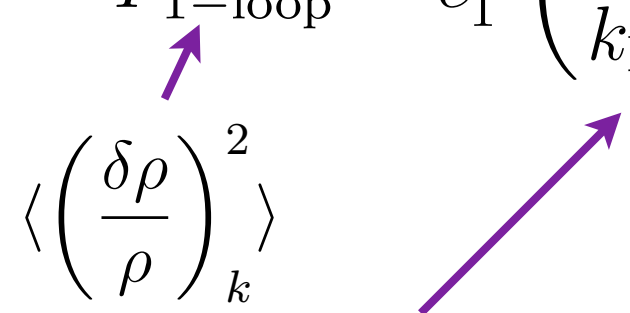
–evaluate with cutoff:

$$\langle \left( \frac{\delta \rho}{\rho} \right)_k^2 \rangle = c_1^\Lambda \left( \frac{\Lambda}{k_{\text{NL}}} \right) \left( \frac{k}{k_{\text{NL}}} \right)^2 P_{11} + c_1^{\text{finite}} \left( \frac{k}{k_{\text{NL}}} \right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\text{NL}}}$$


– divergence (we extrapolated the equations where they were not valid anymore)

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– divergence (we extrapolated the equations where they were not valid anymore)

– we need to add effect of stress tensor  $\tau_{ij} \supset c_s^2 \delta_\ell$

$$P_{11, c_s} = c_s \left( \frac{k}{k_{\text{NL}}} \right)^2 P_{11} \quad , \quad \text{choose} \quad c_s = -c_1^\Lambda \left( \frac{\Lambda}{k_{\text{NL}}} \right) + c_{s, \text{finite}}$$

$$\Rightarrow P_{1-\text{loop}} + P_{11, c_s} = c_{s, \text{finite}} \left( \frac{k}{k_{\text{NL}}} \right)^2 P_{11} + c_1^{\text{finite}} \left( \frac{k}{k_{\text{NL}}} \right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\text{NL}}}$$

–we just re-derived renormalization

–after renormalization, result is finite and small for  $\frac{k}{k_{\text{NL}}} \ll 1$



# Perturbation Theory within the EFT

Pajer and Zaldarriaga 2013

- Regularization and renormalization of loops (no-scale universe)  $P_{11}(k) = \frac{1}{k_{\text{NL}}^3} \left( \frac{k}{k_{\text{NL}}} \right)^n$

–evaluate with cutoff:

$$P_{1-\text{loop}} = c_1^\Lambda \left( \frac{\Lambda}{k_{\text{NL}}} \right) \left( \frac{k}{k_{\text{NL}}} \right)^2 P_{11} + c_1^{\text{finite}} \left( \frac{k}{k_{\text{NL}}} \right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\text{NL}}}$$

$\left\langle \left( \frac{\delta\rho}{\rho} \right)_k^2 \right\rangle$

– divergence (we extrapolated the equations where they were not valid anymore)

– we need to add effect of stress tensor  $\tau_{ij} \supset c_s^2 \delta_\ell$

$$P_{11, c_s} = c_s \left( \frac{k}{k_{\text{NL}}} \right)^2 P_{11}, \text{ choose } c_s = -c_1^\Lambda \left( \frac{\Lambda}{k_{\text{NL}}} \right) + c_{s, \text{finite}}$$

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$\langle \left( \frac{\delta\rho}{\rho} \right)_k^2 \rangle$

- divergence (we extrapolated the equations where they were not valid anymore)
- we need to add effect of stress tensor  $\tau_{ij} \supset c_s^2 \delta_\ell$

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–we just re-derived renormalization

–after renormalization, result is finite and small for  $\frac{k}{k_{\text{NL}}} \ll 1$

.... lots of work ....

# Galaxy Statistics

Senatore **1406**

with Lewandowsky *et al* **1512**

with Perko *et al.* **1610**



- On galaxies, a long history before us, summarized by McDonald, Roy **2010** .
  - Senatore **1406** provided first complete parametrization.

- Nature of Galaxies is very complicated

$$n_{\text{gal}}(x) = f_{\text{very complicated}} \left( \{H, \Omega_m, \dots, m_e, g_{ew}, \dots, \rho(x)\}_{\text{past light cone}} \right)$$

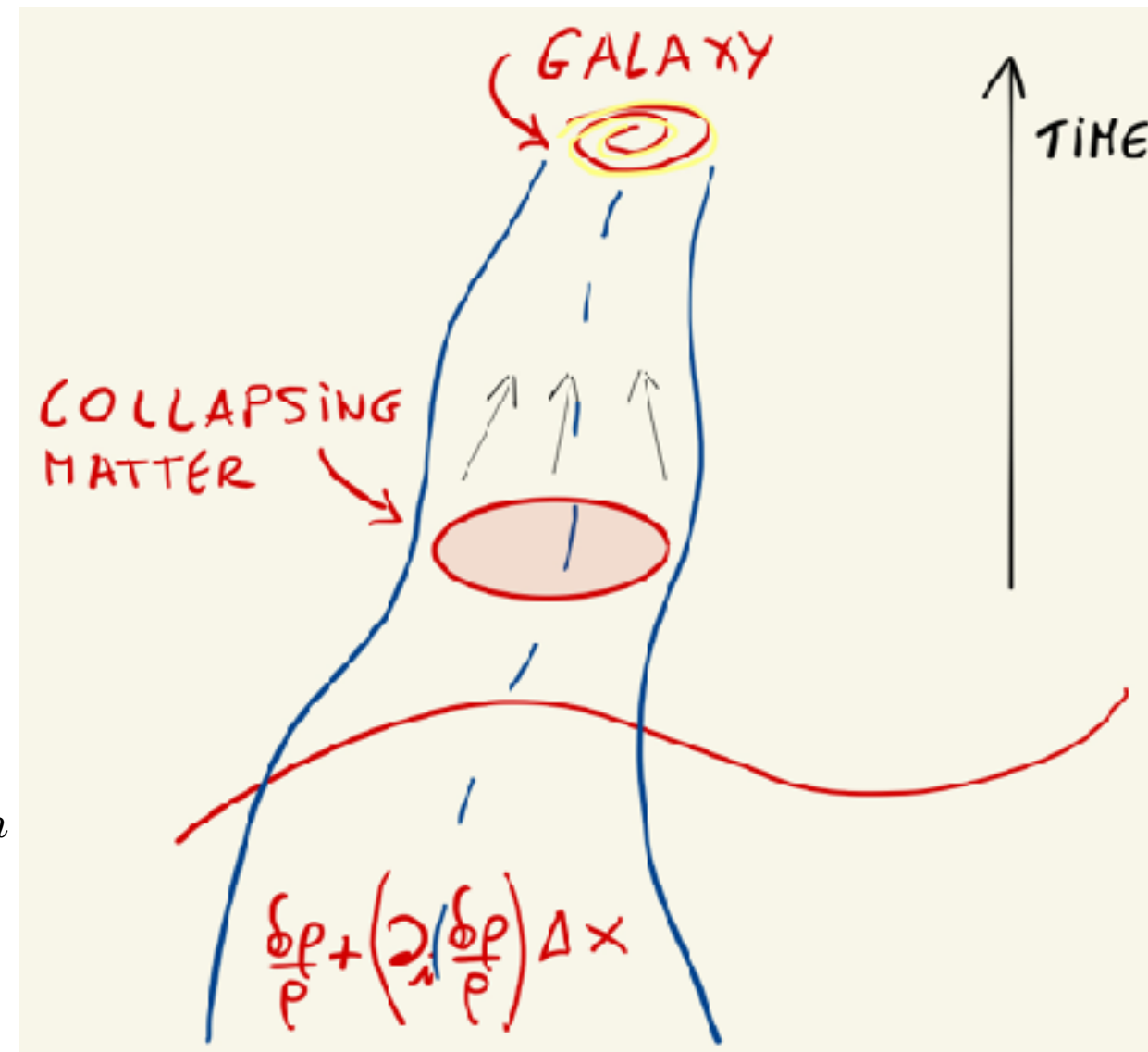
$$n_{\text{gal}}(x) = f_{\text{very complicated}} \left( \{H, \Omega_m, \dots, m_e, g_{ew}, \dots, \rho(x)\}_{\text{past light cone}} \right)$$

At long wavelengths  $\Downarrow$  Taylor Expansion

$$\left( \frac{\delta n}{n} \right)_{\text{gal}, \ell}(x) \sim \int^t dt' \left[ c(t, t') \left( \frac{\delta \rho}{\rho} \right)(\vec{x}_{\text{fl}}, t') + \dots \right]$$

- all terms allowed by symmetries
- all physical effects included  
–e.g. assembly bias

$$\begin{aligned} & \left\langle \left( \frac{\delta n}{n} \right)_{\text{gal}, \ell}(x) \left( \frac{\delta n}{n} \right)_{\text{gal}, \ell}(y) \right\rangle = \\ & = \sum_n \text{Coeff}_n \cdot \langle \text{matter correlation function} \rangle_n \end{aligned}$$



# It is familiar in dielectric E&M

- Polarizability:

$$\vec{P}(\omega) = \chi(\omega) \vec{E}(\omega) \quad \Rightarrow \quad \vec{P}(t) = \int dt' \chi(t - t') \vec{E}(t')$$

- and in fact, also the EFT of Non-Relativistic binaries Goldberger and Rothstein **2004**  
is non-local in time.

# Consequences of non-locality in time

with Carrasco, Foreman, Green **1310**

Senatore **1406**

- The EFT is non-local in time  $\Rightarrow \langle \tau_{ij}(\vec{x}, t) \rangle_{\text{long fixed}} \sim \int^t dt' K(t, t') \delta\rho(\vec{x}_{\text{fl}}, t') + \dots$

- Perturbative Structure has a decoupled structure

$$\delta\rho(x, t') = D(t')\delta\rho(\vec{x})^{(1)} + D(t')^2\delta\rho(\vec{x})^{(2)} + \dots$$

- A few coefficients for each counterterm:

$$\begin{aligned} \Rightarrow \langle \tau_{ij}(\vec{x}, t) \rangle_{\text{long fixed}} &\sim \int^t dt' K(t, t') [D(t')\delta\rho(\vec{x})^{(1)} + D(t')^2\delta\rho(\vec{x})^{(2)} + \dots] \simeq \\ &\simeq c_1(t) \delta\rho(\vec{x})^{(1)} + c_2(t) \delta\rho(\vec{x})^{(2)} + \dots \end{aligned}$$

- where

$$c_i(t) = \int dt' K(t, t') D(t')^i$$

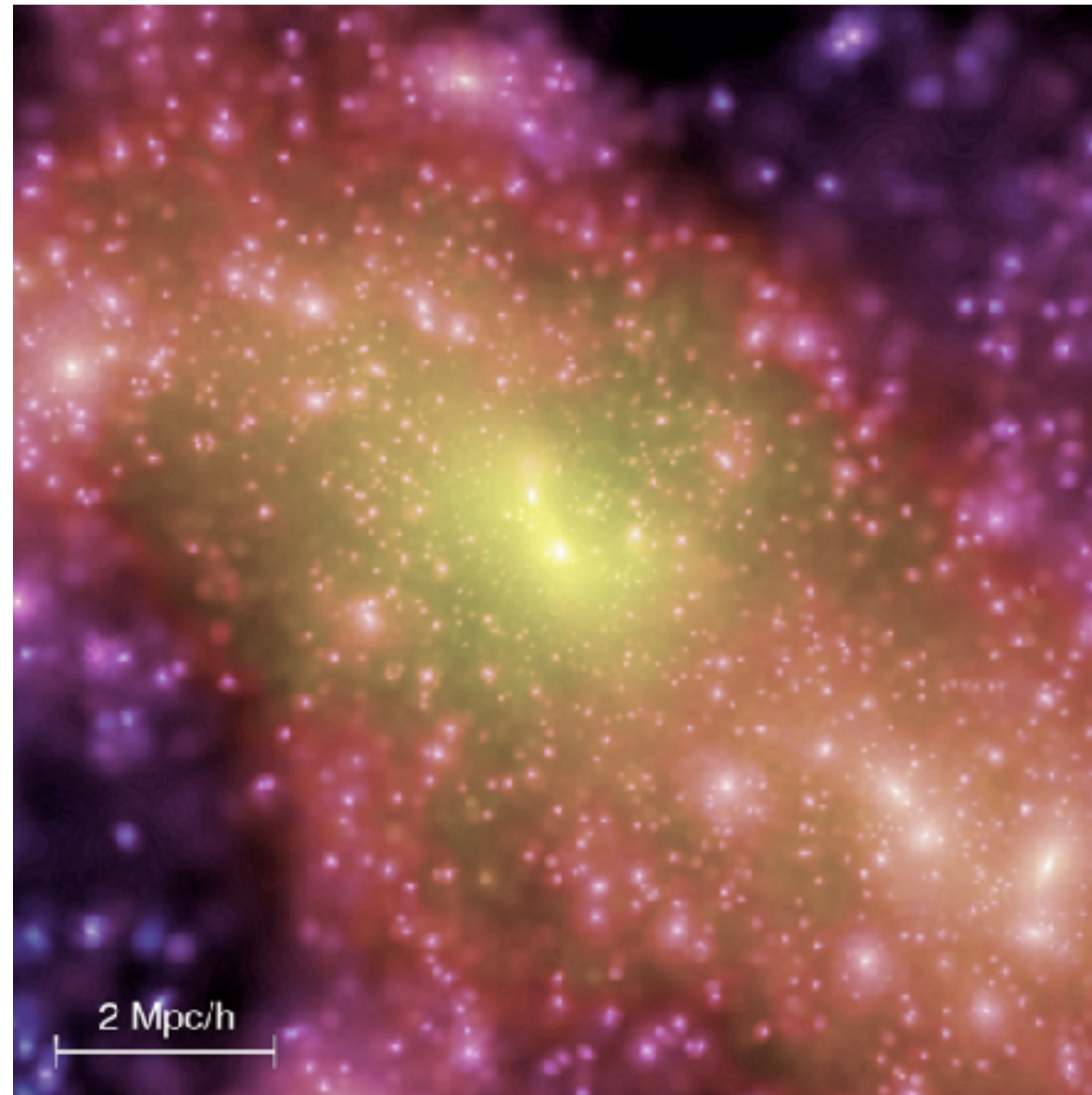
- Difference: Time-Local QFT:  $c_1(t) [\delta\rho(\vec{x})^{(1)} + \delta\rho(\vec{x})^{(2)} + \dots]$   
Non-Time-Local QFT:  $c_1(t) \delta\rho(\vec{x})^{(1)} + c_2(t)\delta\rho(\vec{x})^{(2)} + \dots$

- More terms, but not a disaster



# Baryonic effects

- When stars explode, baryons behave differently than dark matter



credit: Millenium Simulation,  
Springel *et al.* (2005)

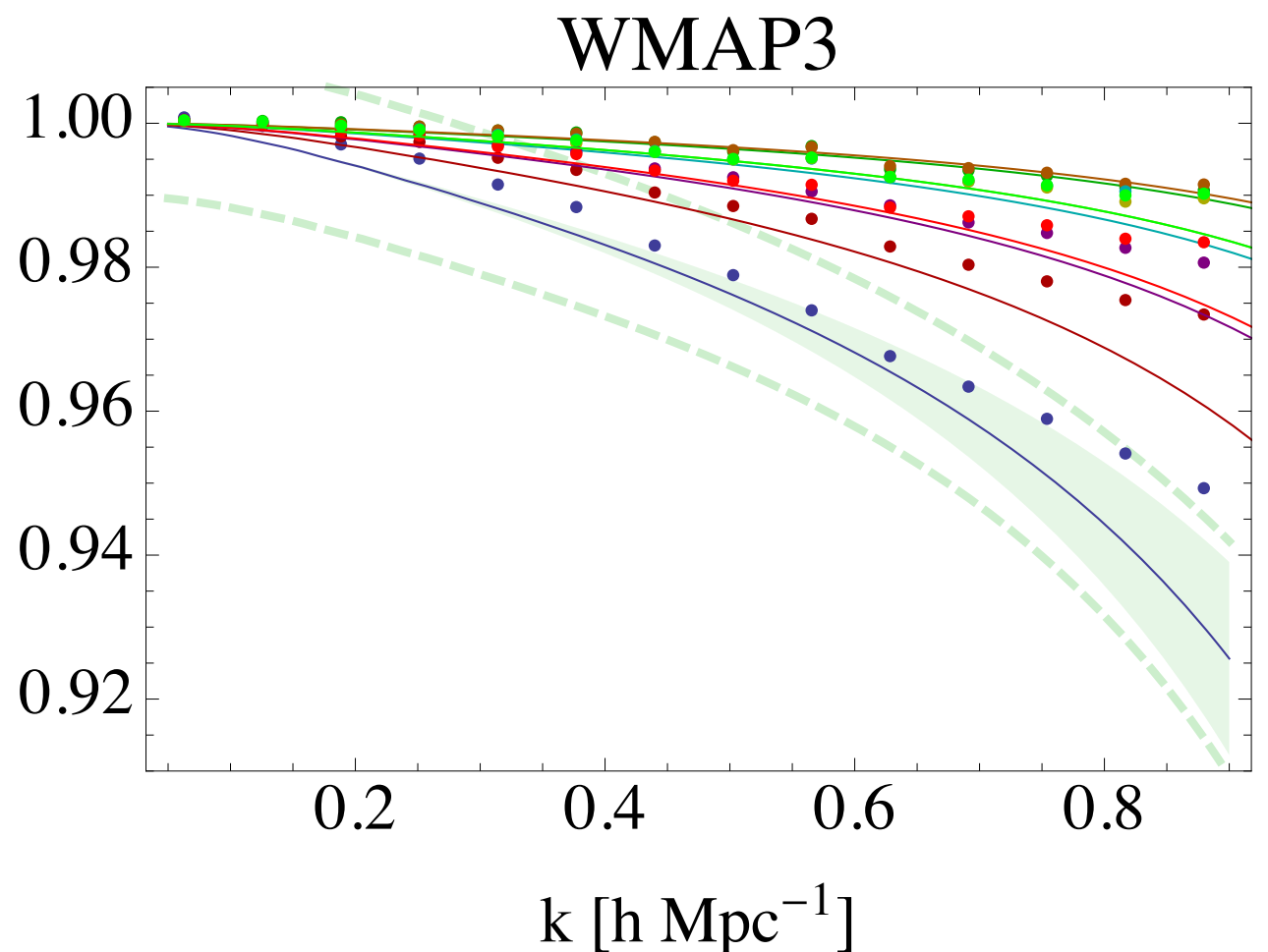
- They cannot be reliably simulated due to large range of scales

# Baryons

- Idea for EFT for dark matter:
  - Dark Matter moves  $1/k_{\text{NL}} \sim 10 \text{ Mpc}$ 
    - $\Rightarrow$  an effective fluid-like system with mean free path  $\sim 1/k_{\text{NL}}$
- Baryons heat due to star formation, but move the same:
  - Universe with CDM+Baryons  $\Rightarrow$  EFTofLSS with 2 specie

$$\Delta P_b(k) \simeq c_\star^2 \left( \frac{k}{k_{\text{NL}}} \right)^2 P_{11}^A(k)$$

$$R = \frac{P^A_{\text{with baryon}}}{P^A_{\text{DM only}}}$$



# Baryons

- EFT Equations:

Continuity:  $\dot{\rho}_\sigma + 3H\rho_\sigma + a^{-1}\partial_i\pi_\sigma^i = 0$  ,

Momentum:  $\dot{\pi}_c^i + 4H\pi_c^i + a^{-1}\partial_j\left(\frac{\pi_c^i\pi_c^j}{\rho_c}\right) + a^{-1}\rho_c\partial_i\Phi = +a^{-1}\gamma^i - a^{-1}\partial_j\tau_c^{ij}$  ,

$$\dot{\pi}_b^i + 4H\pi_b^i + a^{-1}\partial_j\left(\frac{\pi_b^i\pi_b^j}{\rho_b}\right) + a^{-1}\rho_b\partial_i\Phi = -a^{-1}\gamma^i - a^{-1}\partial_j\tau_b^{ij} .$$

# Baryons

- EFT Equations:

Continuity:  $\dot{\rho}_\sigma + 3H\rho_\sigma + a^{-1}\partial_i\pi_\sigma^i = 0$  ,

Momentum:  $\dot{\pi}_c^i + 4H\pi_c^i + a^{-1}\partial_j\left(\frac{\pi_c^i\pi_c^j}{\rho_c}\right) + a^{-1}\rho_c\partial_i\Phi = +a^{-1}\gamma^i - a^{-1}\partial_j\tau_c^{ij}$  ,

$$\dot{\pi}_b^i + 4H\pi_b^i + a^{-1}\partial_j\left(\frac{\pi_b^i\pi_b^j}{\rho_b}\right) + a^{-1}\rho_b\partial_i\Phi = -a^{-1}\gamma^i - a^{-1}\partial_j\tau_b^{ij} .$$

dynamical friction

effective force

- Counterterms:

$$\begin{aligned} \partial_i(\partial\tau_\rho)_c^i - \partial_i(\gamma)_c^i = & -g w_b a \left( H \partial_i v_I^i + 9(2\pi)H^2 \left\{ \frac{c_{c,g}^2}{k_{\text{NL}}^2} (w_c\partial^2\delta_c + w_b\partial^2\delta_b) + \frac{c_{c,v}^2}{k_{\text{NL}}^2} \partial^2\delta_c \right. \right. \\ & + \frac{1}{k_{\text{NL}}^2} \left( c_{1c}^{cc}\partial^2\delta_c^2 + c_{1c}^{cb}\partial^2(\delta_c\delta_b) + c_{1c}^{bb}\partial^2\delta_b^2 \right) \\ & \left. \left. + \frac{c_{4c,g}^2}{a^2 k_{\text{NL}}^4} (w_c\partial^4\delta_c + w_b\partial^4\delta_b) + \frac{c_{4c,v}^2}{a^2 k_{\text{NL}}^4} \partial^4\delta_c \right\} + \dots \right) \end{aligned}$$



# A relevant operator

- Dynamical friction term is indeed needed for renormalization of the theory, i.e. it is generated.
- Dynamical friction is a relevant operator: i.e. it cannot be treated perturbatively: it is an essential part of the linear *equations*:

$$a^2 \delta_I^{(1)''}(a, \vec{k}) + \left( 2 + \frac{a \mathcal{H}'(a)}{\mathcal{H}(a)} \right) a \delta_I^{(1)'}(a, \vec{k}) = \int^a da_1 g(a, a_1) a_1 \delta_I^{(1)'}(a_1, \vec{k}) .$$

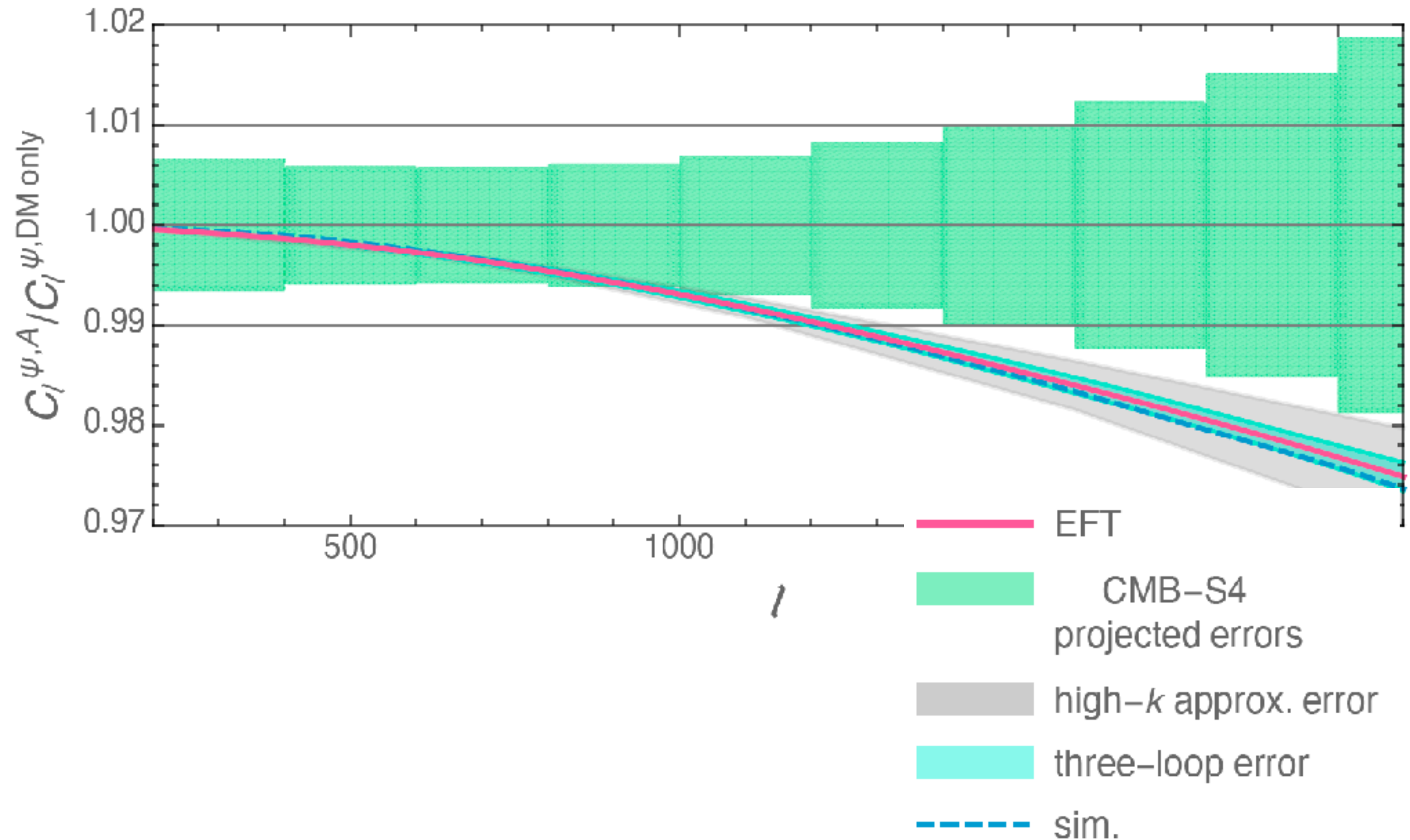
–due to the time-translation breaking and actually even non-locality, **very very very very very very hard** to handle consistently.

- we can make some guesses

- Luckily: it only affect the decaying mode of the isocurvature, which is **very very very very very small**.

# Predictions for CMB Lensing

- Baryon corrections are detectable in next CMB S-4 experiments. But we can predict it:



# Bispectrum at one loop

with D'Amico, Donath, Lewandowski, Zhang **2206**

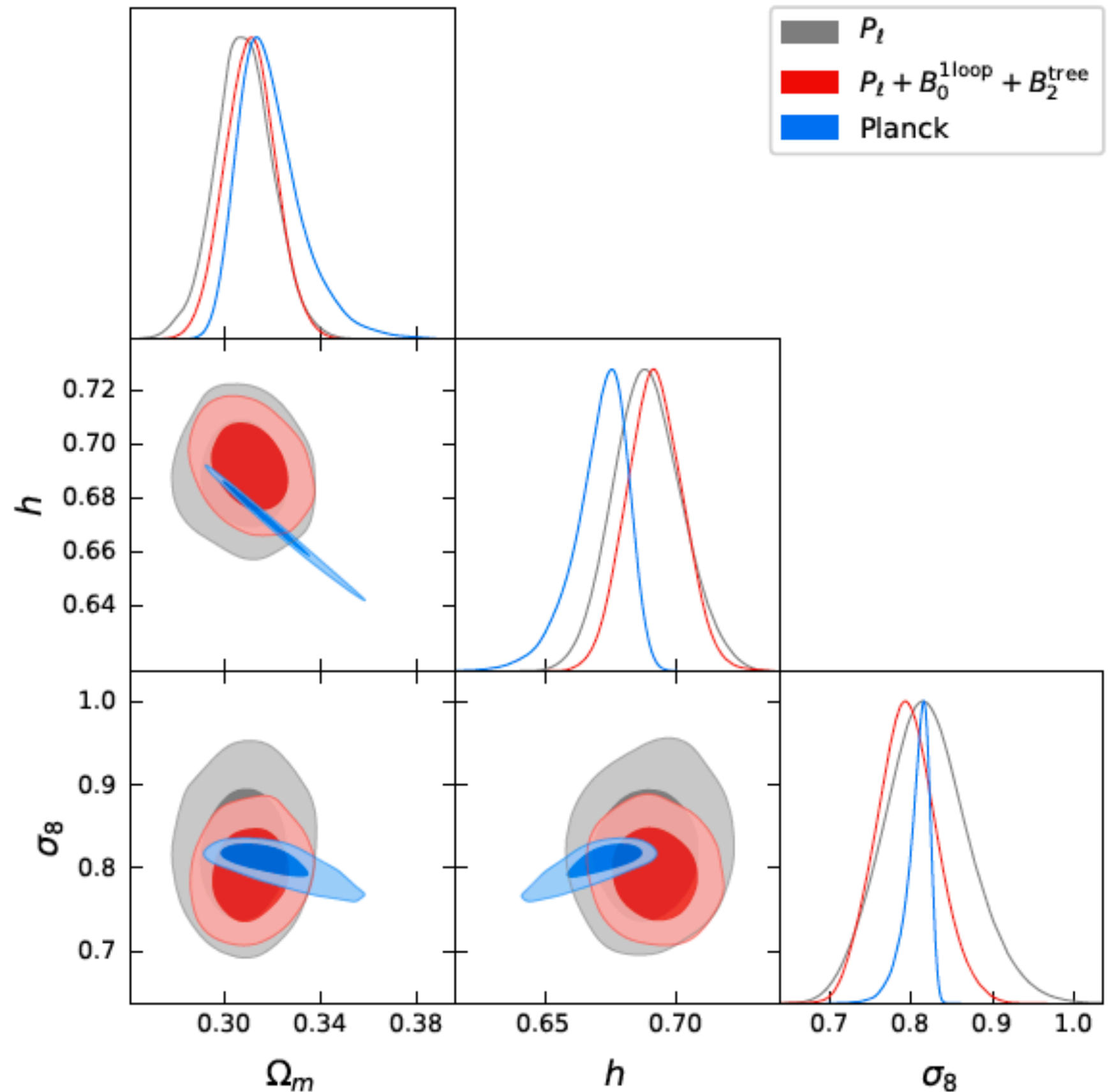
# Bispectrum

- The tree level bispectrum had been already used for cosmological parameter analysis in
  - with Guido D'Amico, Jerome Gleyzes, Nickolas Kockron, Dida Markovic, Pierre Zhang, Florian Beutler, Hector Gill-Marin **1909.05271**
  - Philcox, Ivanov **2112**
- $\sim 10\%$  improvement on  $A_s$
- Time to move to one-loop:
  - Large effort:
    - data analysis with D'Amico, Donath, Lewandowski, Zhang **2206**
    - theory model with D'Amico, Donath, Lewandowski, Zhang **2211**
    - theory integration with Anastasiou, Braganca, Zheng **2212**

# Data Analysis

with D'Amico, Donath, Lewandowski, Zhang **2206**

- Main result:
  - Improvements:
    - 30% on  $\sigma_8$
    - 18% on  $h$
    - 13% on  $\Omega_m$
- Compatible with Planck
  - no tensions



- We add all the relevant biases (4th order) and counterterms (2nd order):

$$P_{11}^{r,h}[b_1] , \quad P_{13}^{r,h}[b_1, b_3, b_8] , \quad P_{22}^{r,h}[b_1, b_2, b_5] , \\ B_{211}^{r,h}[b_1, b_2, b_5] , \quad B_{321}^{r,h,(II)}[b_1, b_2, b_3, b_5, b_8] , \quad B_{411}^{r,h}[b_1, \dots, b_{11}] , \\ B_{222}^{r,h}[b_1, b_2, b_5] , \quad B_{321}^{r,h,(I)}[b_1, b_2, b_3, b_5, b_6, b_8, b_{10}] ,$$

$$P_{13}^{r,h,ct}[b_1, c_{h,1}, c_{\pi,1}, c_{\pi v,1}, c_{\pi v,3}] , \quad P_{22}^{r,h,\epsilon}[c_1^{\text{St}}, c_2^{\text{St}}, c_3^{\text{St}}] , \\ B_{321}^{r,h,(II),ct}[b_1, b_2, b_5, c_{h,1}, c_{\pi,1}, c_{\pi v,1}, c_{\pi v,3}] , \quad B_{321}^{r,h,\epsilon,(I)}[b_1, c_1^{\text{St}}, c_2^{\text{St}}, \{c_i^{\text{St}}\}_{i=4,\dots,13}] , \\ B_{411}^{r,h,ct}[b_1, \{c_{h,i}\}_{i=1,\dots,5}, c_{\pi,1}, c_{\pi,5}, \{c_{\pi v,j}\}_{j=1,\dots,7}] , \quad B_{222}^{r,h,\epsilon}[c_1^{(222)}, c_2^{(222)}, c_5^{(222)}] .$$

- IR-resummation:

- For the power spectrum, we use the correct and controlled IR-resummation.
- For the bispectrum, we use the wiggle/no-wiggle approximation Ivanov and Sibiriyakov **2018**

$$B_{211}^{r,h} = 2K_1^{r,h}(\vec{k}_1; \hat{z})K_1^{r,h}(\vec{k}_2; \hat{z})K_2^{r,h}(\vec{k}_1, \vec{k}_2; \hat{z})P_{\text{LO}}(k_1)P_{\text{LO}}(k_2) + 2 \text{ perms.} ,$$

$$P_{\text{LO}}(k) = P_{\text{nw}}(k) + (1 + k^2 \Sigma_{\text{tot}}^2) e^{-\Sigma_{\text{tot}}^2} P_{\text{w}}(k)$$

- For the loop, we just use  $P_{\text{NLO}}(k) = P_{\text{nw}}(k) + e^{-\Sigma_{\text{tot}}^2} P_{\text{w}}(k)$  , in the non-integrated power spectra

# Derivation of theory model

with D'Amico, Donath, Lewandowski, Zhang

**2211**

- Counterterms: major algebraic effort for 4th order and some theoretical subtle aspects.
- Renormalization of velocity

- In the EFTofLSS, the velocity is a composite operator  $v^i(x) = \frac{\pi^i(x)}{\rho(x)}$ , so, it needs to be renormalized:

$$[v^i]_R = v^i + \mathcal{O}_v^i,$$

- Under a diffeomorphisms:

$$v^i \rightarrow v^i + \chi^i \Rightarrow \mathcal{O}_v^i \text{ is a scalar}$$

- In redshift space, we have local product of velocities, which need to be renormalized but have non-trivial transformations under diff.s:

$$[v^i v^j]_R \rightarrow [v^i v^j]_R + [v^i]_R \chi^j + [v^j]_R \chi^i + \chi^i \chi^j$$

- To achieve this, one can do: (so must include products  $v^i \cdot \mathcal{O}_v^i$ )

$$[v^i v^j]_R = [v^i]_R [v^j]_R + \mathcal{O}_{v^2}^{ij}, \quad \text{where } \mathcal{O}_{v^2}^{ij} \text{ is a scalar}$$



- Counterterms: major algebraic effort for 4th order and some theoretical subtle aspects.
- Non-local-contributing counterterm.
  - This is a normal effect, just strange-looking in the EFTofLSS context.
  - Normally, counterterms are local, but, contributing through non-local Green's functions, they contribute non-locally.

- Counterterms: major algebraic effort for 4th order and some theoretical subtle aspects.
- Non-local-contributing counterterm.

- In the EFTofLSS, the Green's function is simple:  $\frac{1}{\partial^2}$
- Counterterms typically come with  $\partial^2 \mathcal{O}_{\text{local}} \Rightarrow \delta_{\text{counter}} \sim \frac{1}{\partial^2} \partial^2 \mathcal{O}_{\text{local}} \sim \mathcal{O}_{\text{local}}$ 
  - result almost trivial
- But at second order, and for velocity fields, contracted along the line of sight, the derivative do not cancel, so we get

$$\begin{aligned} \delta_{\text{counter}}(\vec{x}) &\sim \hat{z}^i \hat{z}^j \partial_i \pi_{(2)}^j(\vec{x}) \sim \hat{z}^i \hat{z}^j \frac{\partial_i \partial_j \partial_k \partial_m}{\partial^2} \mathcal{O}_{\text{local}} \\ &\sim \hat{z}^i \hat{z}^j \frac{\partial_i \partial_j \partial_k \partial_m}{\partial^2} \left( \frac{\partial_k \partial_l}{H^2} \Phi(\vec{x}) \frac{\partial_l \partial_m}{H^2} \Phi(\vec{x}) \right) \end{aligned}$$

- This is truly non-locally contributing, truly non-trivial.
- We check that all these terms are *needed and sufficient* for renormalization

# Evaluational/Computational Challenge

with Anastasiou, Braganca, Zheng **2212**

# The best approach so far

Simonovic, Baldauf, Zaldarriaga,  
Carrasco, Kollmeier **2018**

- Nice trick for fast evaluation of the loops integrals
- The power spectrum is a numerically computed function
- Decompose linear power spectrum

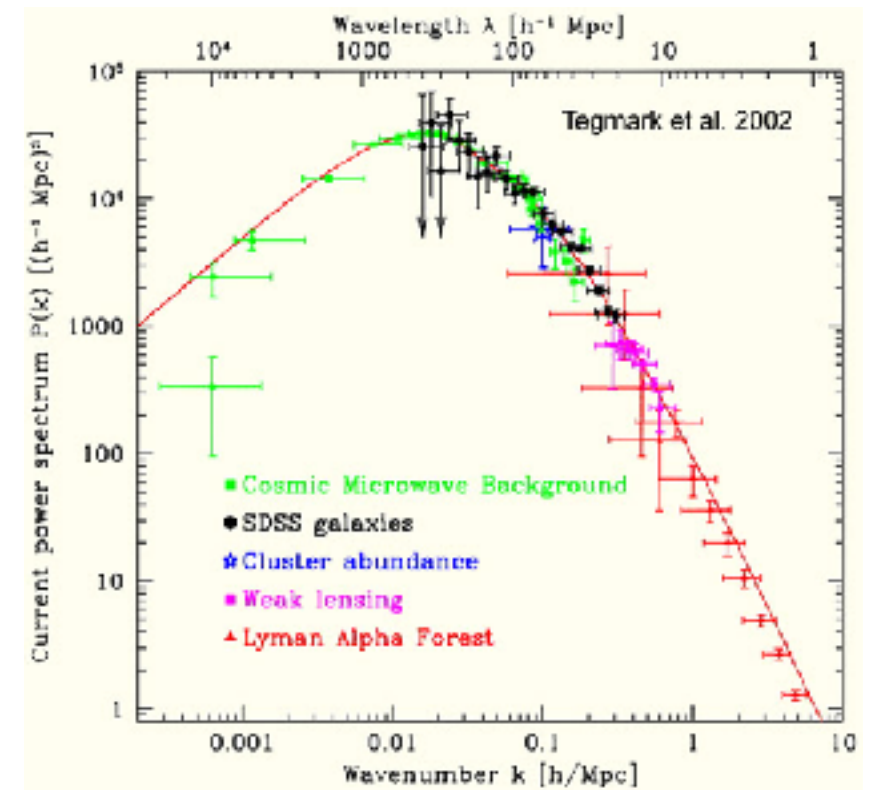
$$P_{11}(k) = \sum_n c_n k^{\mu+i\alpha n}$$

- Loop can be evaluated analytically

$$\begin{aligned} P_{1-\text{loop}}(k) &= \int_{\vec{q}} K(\vec{q}, \vec{k}) P_{11}(k - q) P_{11}(q) = \\ &= \sum_{n_1, n_2} c_{n_1} c_{n_2} \left( \int_{\vec{q}} K(\vec{q}, \vec{k}) k^{\mu+i\alpha n_1} k^{\mu+i\alpha n_2} \right) = \sum_{n_1, n_2} c_{n_1} c_{n_2} M_{n_1, n_2}(k) \end{aligned}$$

–using quantum field theory techniques

–  $M_{n_1 n_2}$  is cosmology independent  $\Rightarrow$  so computed once



- Two difficulties:

$$\begin{aligned} P_{1\text{-loop}}(k) &= \int_{\vec{q}} K(\vec{q}, \vec{k}) P_{11}(k - q) P_{11}(q) = \\ &= \sum_{n_1, n_2} c_{n_1} c_{n_2} \left( \int_{\vec{q}} K(\vec{q}, \vec{k}) k^{\mu+i\alpha n_1} k^{\mu+i\alpha n_2} \right) = \sum_{n_1, n_2} c_{n_1} c_{n_2} M_{n_1, n_2}(k) \end{aligned}$$

- integrals are complicated due to fractional, complex exponents
- many functions needed, the matrix  $M_{n_1 n_2 n_3}$  for bispectrum is about 50Gb, so, ~impossible to load on CPT for data analysis
- In order to ameliorate (solve) these issues, we use a different basis of functions.

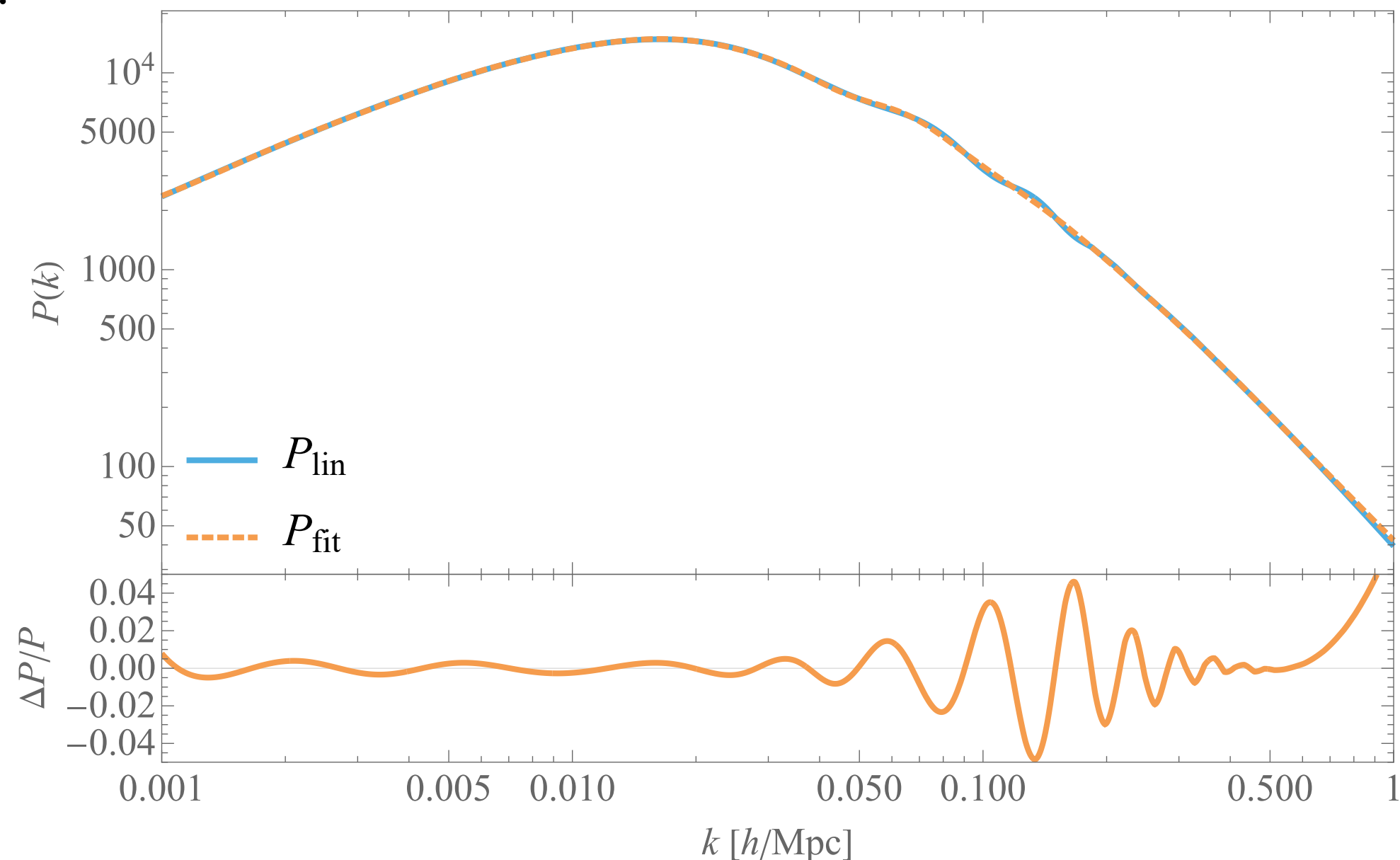
# Complex-Masses Propagators

with Anastasiou, Braganca, Zheng  
2212

- Use as basis:

$$f(k^2, k_{\text{peak}}^2, k_{\text{UV}}^2, i, j) \equiv \frac{(k^2/k_0^2)^i}{\left(1 + \frac{(k^2 - k_{\text{peak}}^2)^2}{k_{\text{UV}}^4}\right)^j},$$

- With just 16 functions:



- This basis is equivalent to massive propagators to integer powers

$$\frac{1}{\left(1 + \frac{(k^2 - k_{\text{peak}}^2)^2}{k_{\text{UV}}^4}\right)^j} = \frac{k_{\text{UV}}^{4j}}{\left(k^2 - k_{\text{peak}}^2 - i k_{\text{UV}}^2\right)^j \left(k^2 - k_{\text{peak}}^2 + i k_{\text{UV}}^2\right)^j},$$

$$\frac{k_{\text{UV}}^2}{\left(k^2 - k_{\text{peak}}^2 - i k_{\text{UV}}^2\right) \left(k^2 - k_{\text{peak}}^2 + i k_{\text{UV}}^2\right)} = -\frac{i/2}{k^2 - k_{\text{peak}}^2 - i k_{\text{UV}}^2} + \frac{i/2}{k^2 - k_{\text{peak}}^2 + i k_{\text{UV}}^2}.$$

- So, each basis function:

$$f(k^2, k_{\text{peak}}^2, k_{\text{UV}}^2, i, j) = \sum_{n=1}^j k_{\text{UV}}^{2(n-i)} k^{2i} \left( \frac{\kappa_n}{(k^2 + M)^n} + \frac{\kappa_n^*}{(k^2 + M^*)^n} \right)$$

- This basis is equivalent to massive propagators to integer powers

$$\frac{1}{\left(1 + \frac{(k^2 - k_{\text{peak}}^2)^2}{k_{\text{UV}}^4}\right)^j} = \frac{k_{\text{UV}}^{4j}}{\left(k^2 - k_{\text{peak}}^2 - i k_{\text{UV}}^2\right)^j \left(k^2 - k_{\text{peak}}^2 + i k_{\text{UV}}^2\right)^j},$$

$$\frac{k_{\text{UV}}^2}{\left(k^2 - k_{\text{peak}}^2 - i k_{\text{UV}}^2\right) \left(k^2 - k_{\text{peak}}^2 + i k_{\text{UV}}^2\right)} = -\frac{i/2}{k^2 - k_{\text{peak}}^2 - i k_{\text{UV}}^2} + \frac{i/2}{k^2 - k_{\text{peak}}^2 + i k_{\text{UV}}^2}.$$

Complex-Mass propagator

- So, each basis function:

$$f(k^2, k_{\text{peak}}^2, k_{\text{UV}}^2, i, j) = \sum_{n=1}^j k_{\text{UV}}^{2(n-i)} k^{2i} \left( \frac{\kappa_n}{(k^2 + M)^n} + \frac{\kappa_n^*}{(k^2 + M^*)^n} \right)$$



- We end up with integral like this:

$$L(n_1, d_1, n_2, d_2, n_3, d_3) = \int_q \frac{(\mathbf{k}_1 - \mathbf{q})^{2n_1} \mathbf{q}^{2n_2} (\mathbf{k}_2 + \mathbf{q})^{2n_3}}{((\mathbf{k}_1 - \mathbf{q})^2 + M_1)^{d_1} (\mathbf{q}^2 + M_2)^{d_2} ((\mathbf{k}_2 + \mathbf{q})^2 + M_3)^{d_3}}$$

- with integer exponents.
- First we manipulate the numerator to reduce to:

$$T(d_1, d_2, d_3) = \int_q \frac{1}{((\mathbf{k}_1 - \mathbf{q})^2 + M_1)^{d_1} (\mathbf{q}^2 + M_2)^{d_2} ((\mathbf{k}_2 + \mathbf{q})^2 + M_3)^{d_3}},$$

- Then, by integration by parts, we find (i.e. QCD teaches us how to) recursion relations

$$\int_q \frac{\partial}{\partial q_\mu} \cdot (q_\mu t(d_1, d_2, d_3)) = 0$$

$$\Rightarrow (3 - d_{1223})\hat{0} + d_1 k_{1s} \hat{1}^+ + d_3 (k_{2s}) \hat{3}^+ + 2M_2 d_2 \hat{2}^+ - d_1 \hat{1}^+ \hat{2}^- - d_3 \hat{2}^- \hat{3}^+ = 0$$

- relating same integrals with raised or lowered the exponents (easy terminate due to integer exponents).

# Complex-Masses Propagators

with Anastasiou, Braganca, Zheng  
2212

- We end up to three master integrals:

- Tadpole:

$$\text{Tad}(M_j, n, d) = \int \frac{d^3 \mathbf{q}}{\pi^{3/2}} \frac{(\mathbf{p}_i^2)^n}{(\mathbf{p}_i^2 + M_j)^d}$$

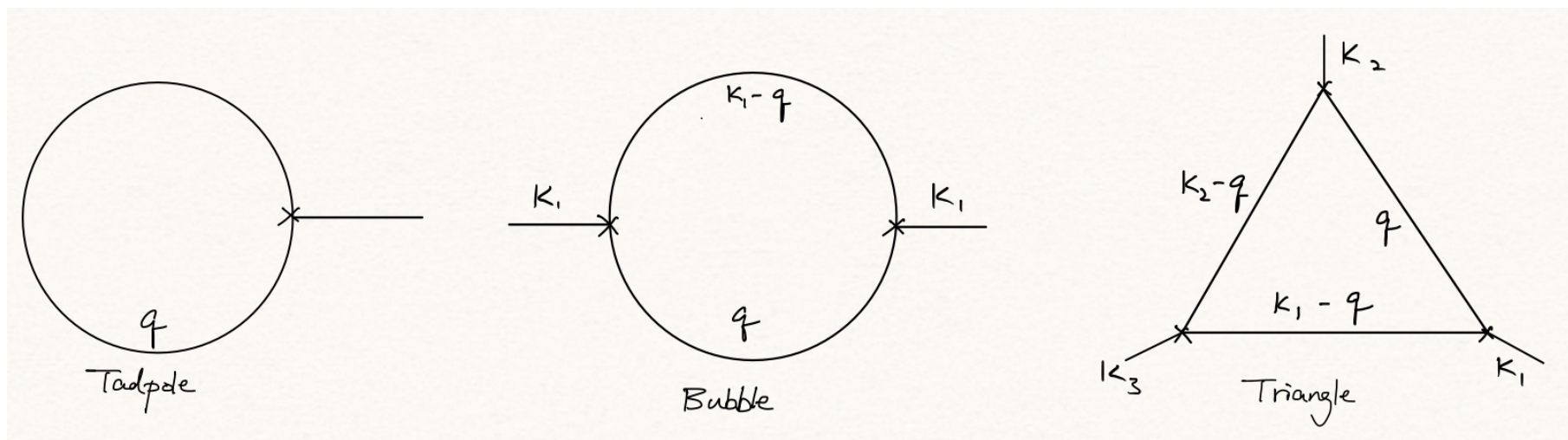
- Bubble:

$$B_{\text{master}}(k^2, M_1, M_2) = \int \frac{d^3 \mathbf{q}}{\pi^{3/2}} \frac{1}{(q^2 + M_1)(|\mathbf{k} - \mathbf{q}|^2 + M_2)}$$

- Triangle:

$$T_{\text{master}}(k_1^2, k_2^2, k_3^2, M_1, M_2, M_3) =$$

$$\int \frac{d^3 \mathbf{q}}{\pi^{3/2}} \frac{1}{(q^2 + M_1)(|\mathbf{k}_1 - \mathbf{q}|^2 + M_2)(|\mathbf{k}_2 + \mathbf{q}|^2 + M_3)},$$



- The master integrals are evaluated with Feynman parameters, but with great care of branch cut crossing, which happens because of complex masses.

- Bubble Master:

$$B_{\text{master}}(k^2, M_1, M_2) = \frac{\sqrt{\pi}}{k} i [\log(A(1, m_1, m_2)) - \log(A(0, m_1, m_2)) - 2\pi i H(\text{Im } A(1, m_1, m_2)) H(-\text{Im } A(0, m_1, m_2))] ,$$

$$A(0, m_1, m_2) = 2\sqrt{m_2} + i(m_1 - m_2 + 1) ,$$

$$A(1, m_1, m_2) = 2\sqrt{m_1} + i(m_1 - m_2 - 1) ,$$

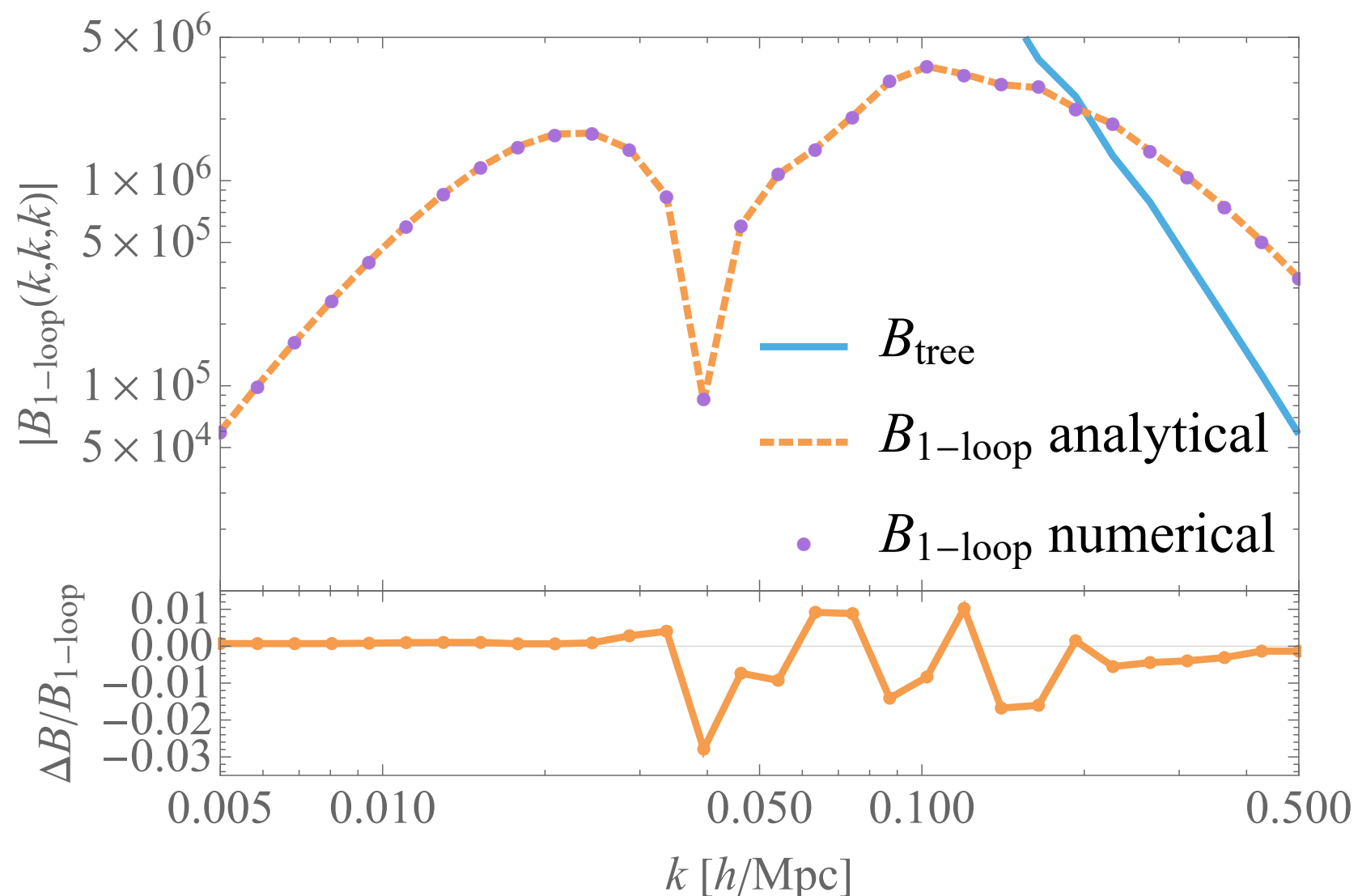
$$m_1 = M_1/k^2 \text{ and } m_2 = M_2/k^2$$

- Triangle Master:

$$F_{\text{int}}(R_2, z_+, z_-, x_0) = s(z_+, -z_-) \frac{\sqrt{\pi}}{\sqrt{|R_2|}} \frac{\arctan\left(\frac{\sqrt{z_+ - x} \sqrt{x_0 - z_-}}{\sqrt{x_0 - z_+} \sqrt{x - z_-}}\right)}{\sqrt{x_0 - z_+} \sqrt{x_0 - z_-}} \bigg|_{x=0}^{x=1} .$$

- Very simple expressions with simple rule for branch cut crossing.

- All automatically coded up.
- For BOSS analysis, evaluation of matrix is 2.5CPU hours and 800 Mb storage, very fast matrix contractions.
- Accuracy with 16 functions:



# Back to data-analysis: Pipeline Validation

# Measuring and fixing phase space

- We consider synthetic data, i.e. data made out of the model, and analyze them:

- Green: biased.

- Why?

- Priors centered on zero?

- Grey: biased

- Bug in pipeline?

- Test by reducing covar.

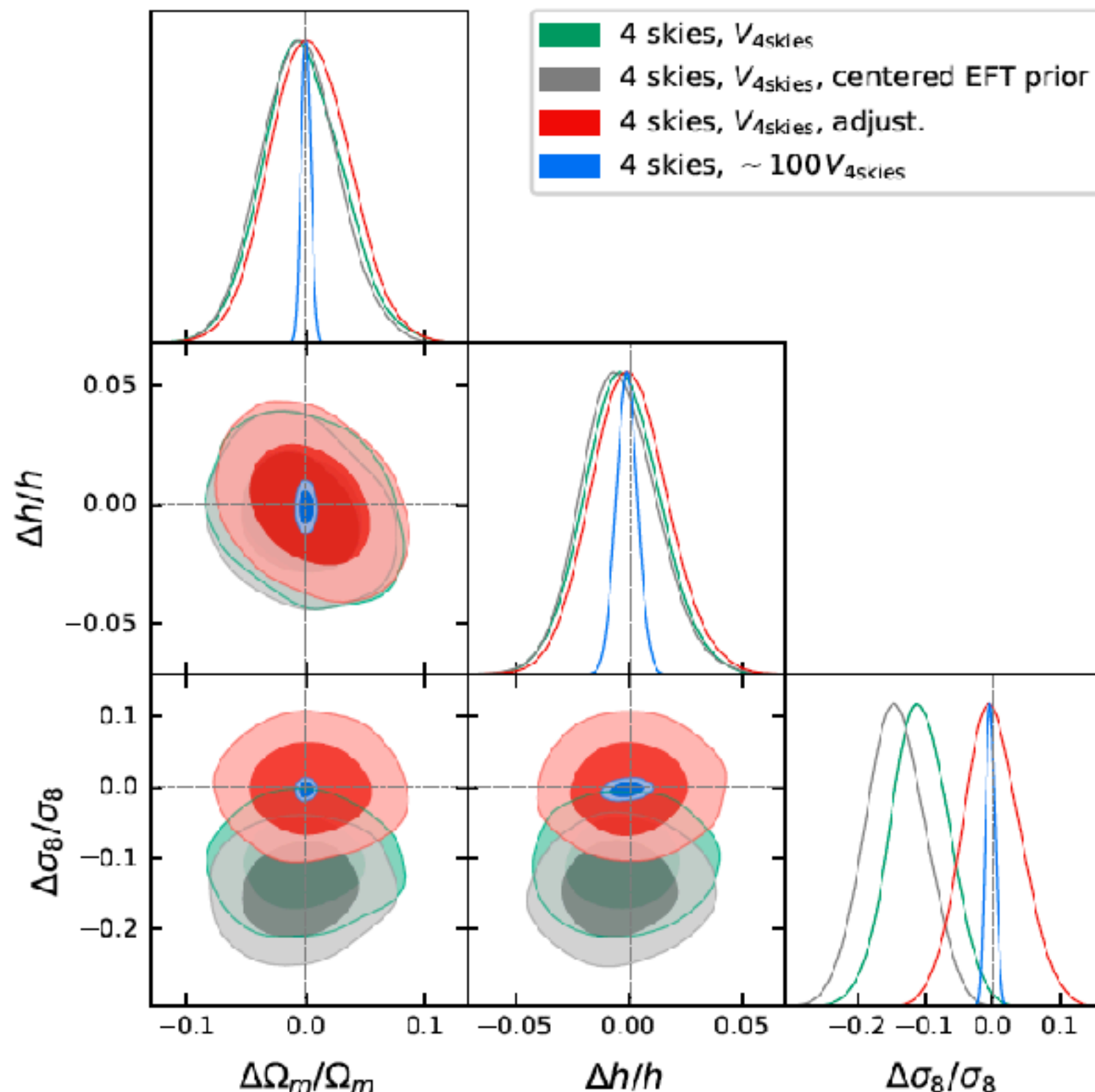
- Red: non-biased

- It must be phase space projection

- But the grey line offers

- an honest measurement of it.

- 



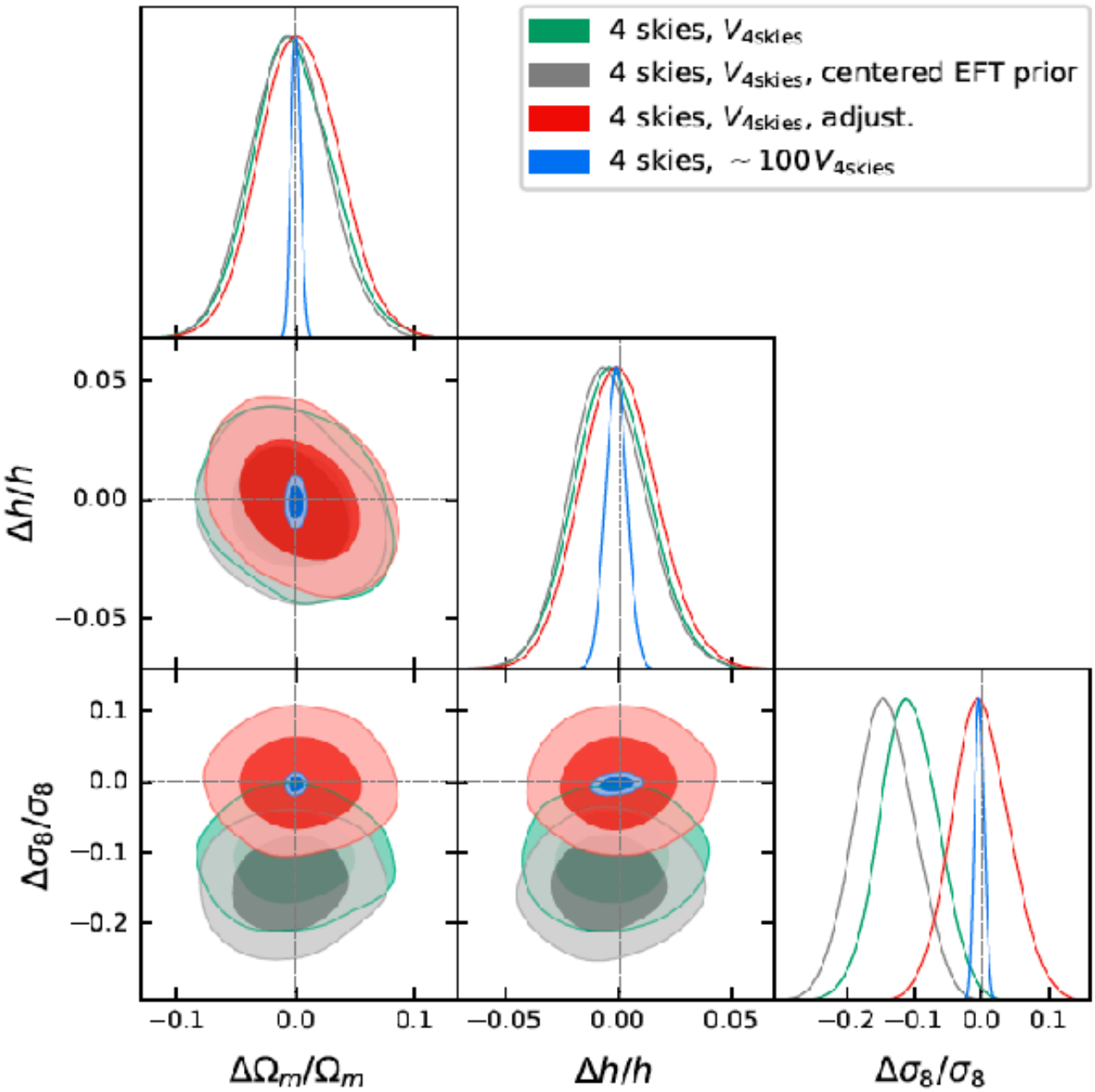
# Measuring and fixing phase space

• We add:

$$\ln \mathcal{P}_{\text{pr}}^{\text{ph. sp. 4sky}} = -48 \left( \frac{b_1}{2} \right) + 32 \left( \frac{\Omega_m}{0.31} \right) + 48 \left( \frac{h}{0.68} \right) \quad ,$$

$\sigma_{\text{proj}}/\sigma_{\text{stat}}$	$\Omega_m$	$h$	$\sigma_8$	$\omega_{\text{cdm}}$
1 sky, $\sim 100 V_{1\text{sky}}$	-0.1	-0.14	-0.21	-0.2
1 sky, $V_{1\text{sky}}$ , adjust.	0.13	0.06	0.04	0.15
4 skies, $V_{4\text{skies}}$ , adjust.	0.1	0.	-0.05	0.07

• no more proj. effect.





- We can estimate the  $k_{\text{max}}$  without the use of simulations, by adding NNLO terms, and seeing when they make a difference on the posteriors.

$$P_{\text{NNLO}}(k, \mu) = \frac{1}{4} c_{r,4} b_1^2 \mu^4 \frac{k^4}{k_{\text{NL,R}}^4} P_{11}(k) + \frac{1}{4} c_{r,6} b_1 \mu^6 \frac{k^4}{k_{\text{NL,R}}^4} P_{11}(k) ,$$

$$\begin{aligned} B_{\text{NNLO}}(k_1, k_2, k_3, \mu, \phi) = & 2 c_{\text{NNLO},1} K_2^{r,h}(\vec{k}_1, \vec{k}_2; \hat{z}) K_1^{r,h}(\vec{k}_2; \hat{z}) f \mu_1^2 \frac{k_1^4}{k_{\text{NL,R}}^4} P_{11}(k_1) P_{11}(k_2) \\ & + c_{\text{NNLO},2} K_1^{r,h}(\vec{k}_1; \hat{z}) K_1^{r,h}(\vec{k}_2; \hat{z}) P_{11}(k_1) P_{11}(k_2) f \mu_3 k_3 \frac{(k_1^2 + k_2^2)}{4 k_1^2 k_2^2 k_{\text{NL,R}}^4} \left[ - 2 \vec{k}_1 \cdot \vec{k}_2 (k_1^3 \mu_1 + k_2^3 \mu_2) \right. \\ & \left. + 2 f \mu_1 \mu_2 \mu_3 k_1 k_2 k_3 (k_1^2 + k_2^2) \right] + \text{perm.} , \end{aligned} \quad (4)$$

- For our  $k_{\text{max}}$ , we find the following shifts, which are ok:

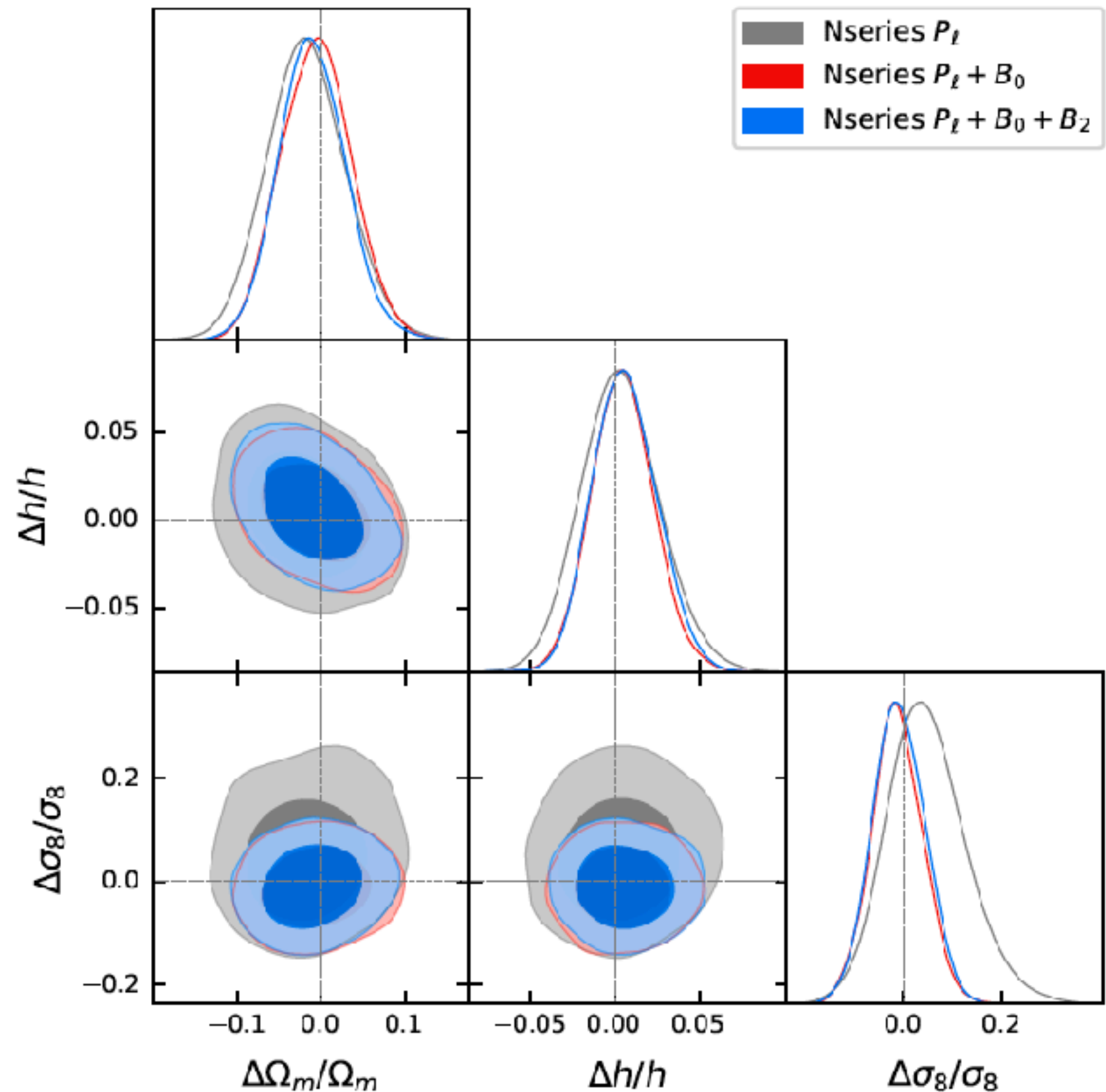
$\Delta_{\text{shift}}/\sigma_{\text{stat}}$	$\Omega_m$	$h$	$\sigma_8$	$\omega_{\text{cdm}}$	$\ln(10^{10} \Lambda_s)$	$S_8$
$P_\ell + B_0$ : base - w/ NNLO	-0.03	-0.09	-0.03	-0.1	0.05	-0.04



# Scale-cut from simulations

with D'Amico, Donath, Lewandowski, Zhang 2206

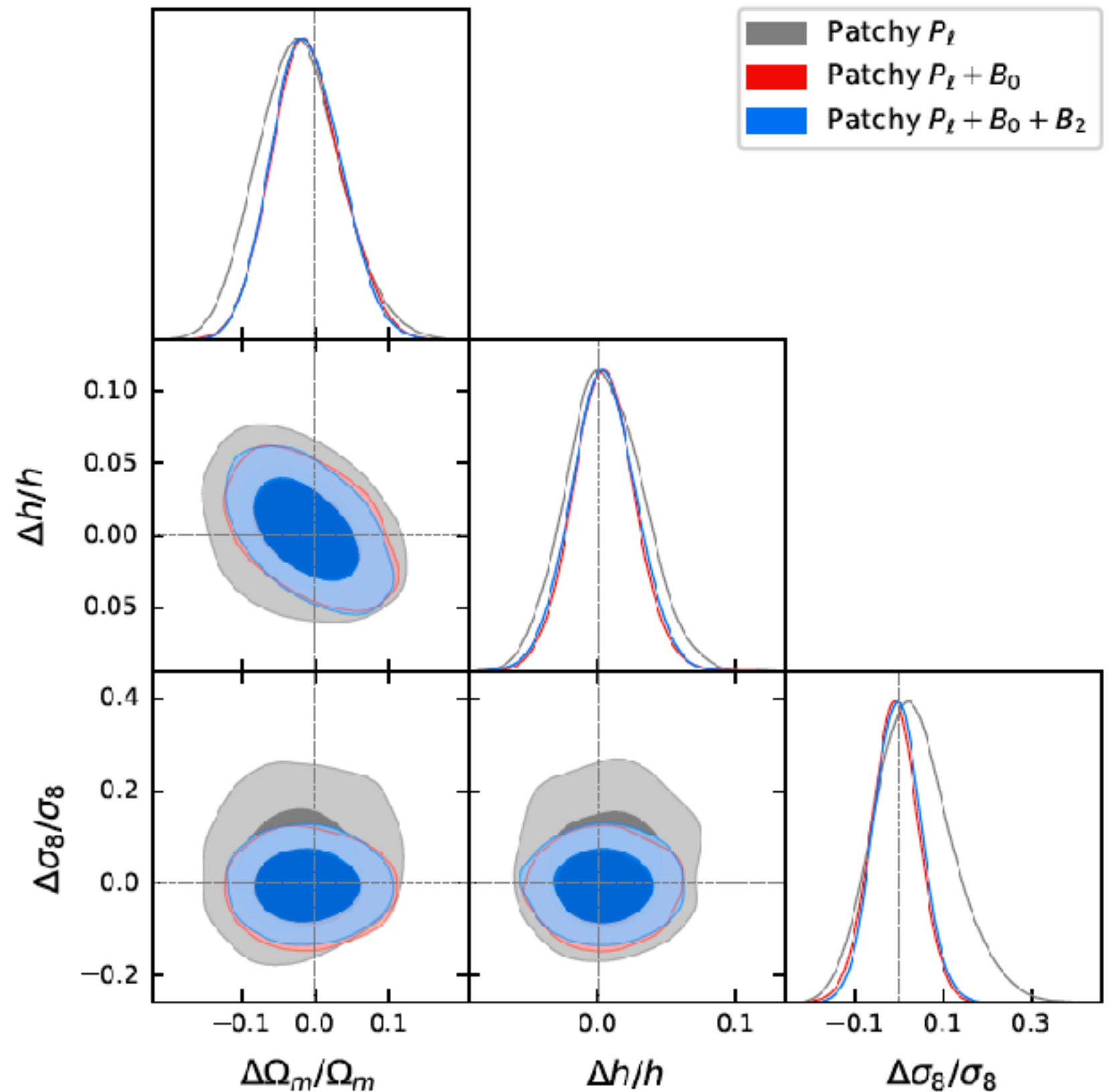
- N-series
  - Volume  $\sim 80$  BOSS
  - safely within  $\sigma_{\text{data}}/3$
- After phase-space correction



# Scale-cut from simulations

with D'Amico, Donath, Lewandowski, Zhang 2206

- Patchy:
  - Volume  $\sim 2000$  BOSS
  - safely within  $\sigma_{\text{data}}/3$
- After phase-space correction

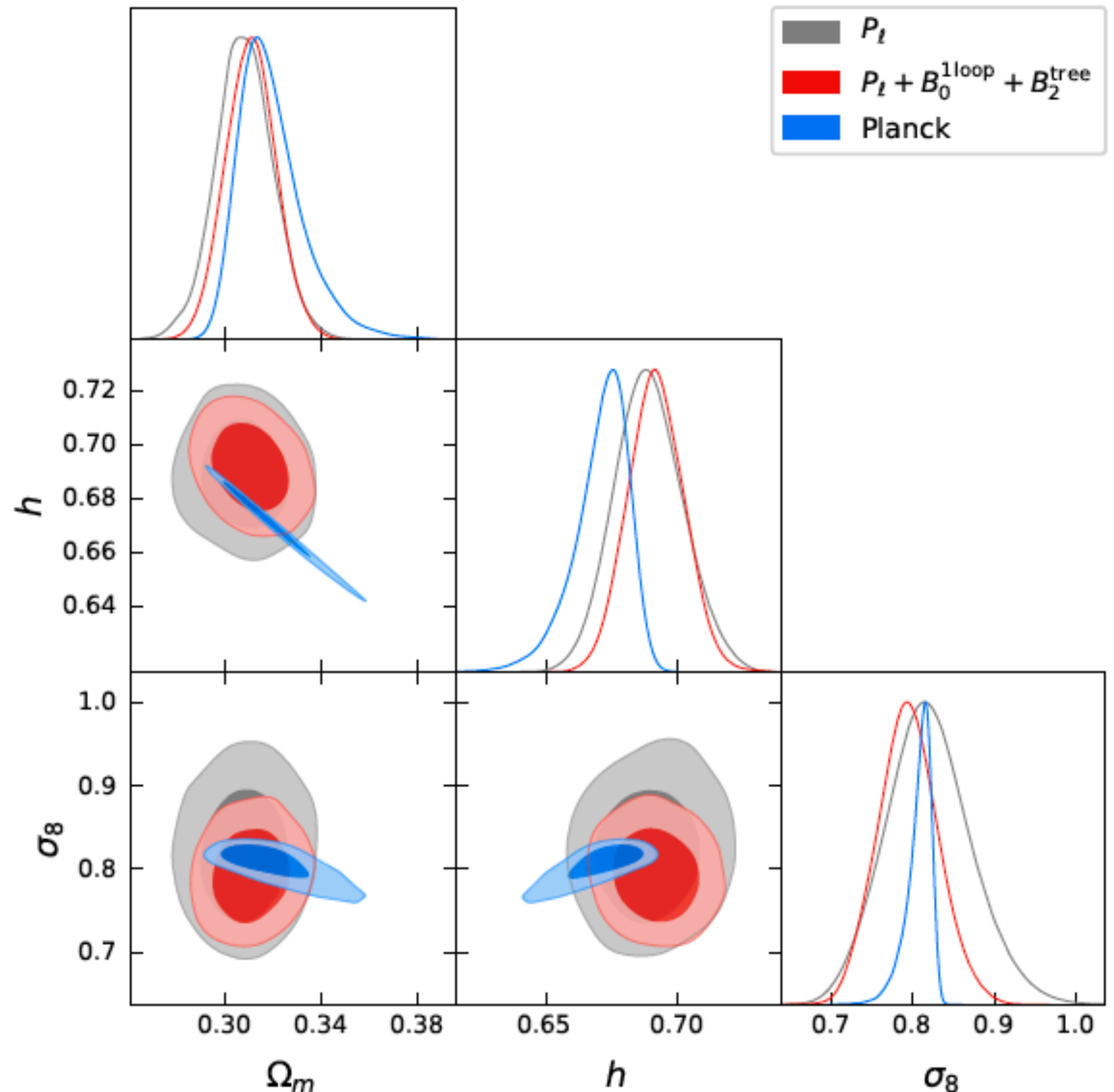


BOSS data

# Data Analysis

with D'Amico, Donath, Lewandowski, Zhang **2206**

- Main result:
  - Improvements:
    - 30% on  $\sigma_8$
    - 18% on  $h$
    - 13% on  $\Omega_m$
- Compatible with Planck
  - no tensions
- Remarkable consistency
  - of observables



# Direct Measurement of formation time of galaxies

with Donath and Lewandowski **to appear**

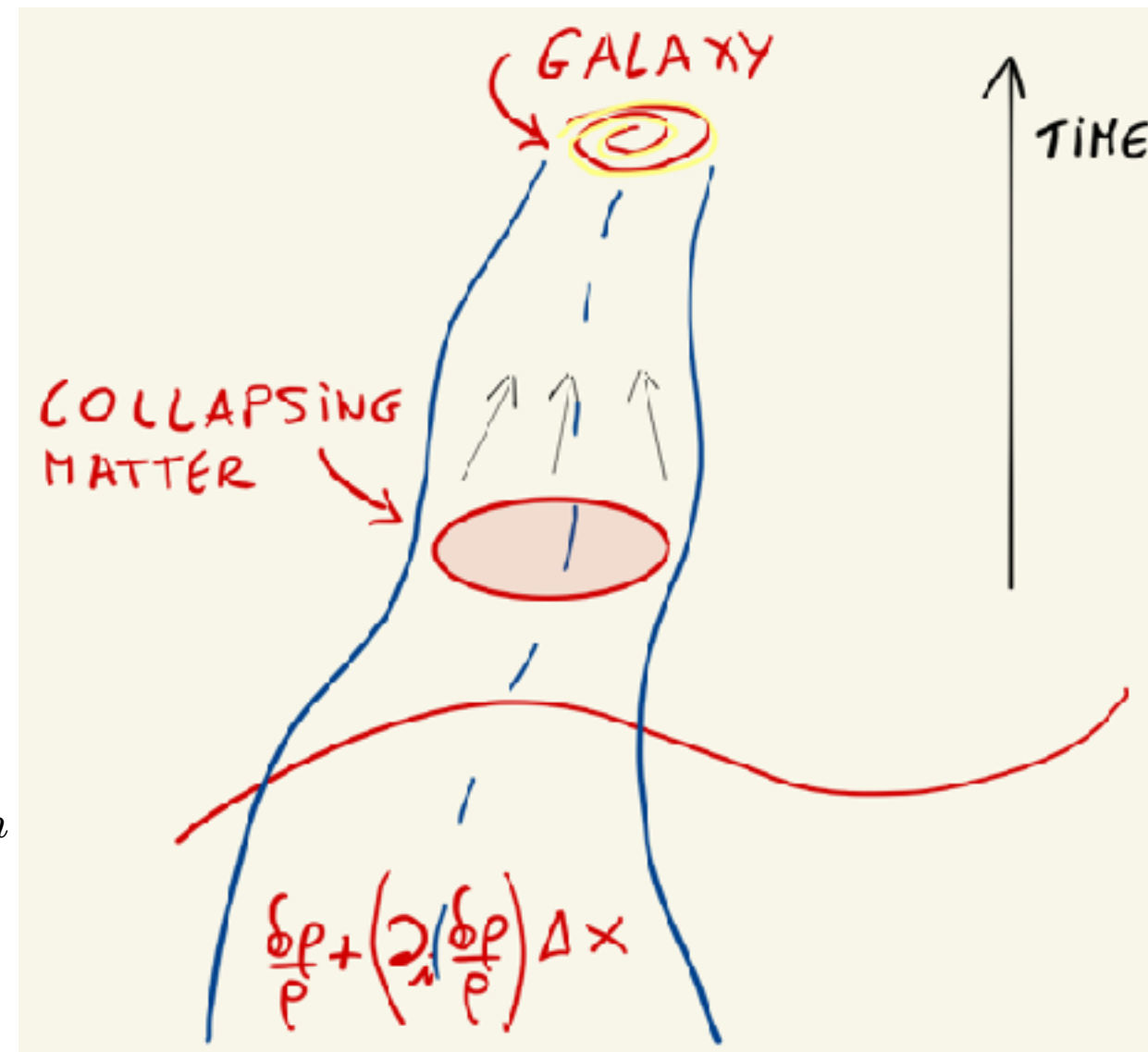
$$n_{\text{gal}}(x) = f_{\text{very complicated}} \left( \{H, \Omega_m, \dots, m_e, g_{ew}, \dots, \rho(x)\}_{\text{past light cone}} \right)$$

At *long* wavelengths  $\Downarrow$  Taylor Expansion

$$\left( \frac{\delta n}{n} \right)_{\text{gal}, \ell}(x) \sim \int^t dt' \left[ c(t, t') \left( \frac{\delta \rho}{\rho} \right)(\vec{x}_{\text{fl}}, t') + \dots \right]$$

- all terms allowed by symmetries
- all physical effects included  
–e.g. assembly bias

$$\begin{aligned} & \left\langle \left( \frac{\delta n}{n} \right)_{\text{gal}, \ell}(x) \left( \frac{\delta n}{n} \right)_{\text{gal}, \ell}(y) \right\rangle = \\ & = \sum_n \text{Coeff}_n \cdot \langle \text{matter correlation function} \rangle_n \end{aligned}$$



# Consequences of non-locality in time

- This means that one *does not* get the same terms as in the local-in-time expansion
- If we could measure one of these terms, we could *measure* that Galaxies take an Hubble time to form. We have never measured this: we take pictures of different galaxies at different stages of their evolution. But we have never *seen* a galaxy form in an Hubble time.
  - This would be the first direct evidence that the universe lasted an Hubble time.
- So, detecting a non-local-in-time bias would allow us to measure that, and from the size, the formation time. Unfortunately, so far, not yet.

# Consequences of non-locality in time

- Mathematics again:

- non-local in time:

$$\delta_g^{(n)}(\vec{x}, t) = \sum_{\mathcal{O}_m} \int^t dt' H(t') c_{\mathcal{O}_m}(t, t') \times [\mathcal{O}_m(\vec{x}_{\text{fl}}(\vec{x}, t, t'), t')]^{(n)},$$

$$\mathcal{O}_{m=3} \supset \delta^2 \theta, \delta^3, \dots$$

- local in time:

$$\Rightarrow \delta_{g, \text{loc}}^{(n)}(\vec{x}, t) = \sum_{\mathcal{O}_m} c_{\mathcal{O}_m}(t) \mathcal{O}_m^{(n)}(\vec{x}, t),$$

- more non local in time:

$$[\mathcal{O}_m(\vec{x}_{\text{fl}}(\vec{x}, t, t'), t')]^{(n)} = \sum_{\alpha=1}^{n-m+1} \left( \frac{D(t')}{D(t)} \right)^{\alpha+m-1} \mathbb{C}_{\mathcal{O}_m, \alpha}^{(n)}(\vec{x}, t)$$

$$\Rightarrow \delta_g^{(n)}(\vec{x}, t) = \sum_{\mathcal{O}_m} \sum_{\alpha=1}^{n-m+1} c_{\mathcal{O}_m, \alpha}(t) \mathbb{C}_{\mathcal{O}_m, \alpha}^{(n)}(\vec{x}, t)$$



# Consequences of non-locality in time

$$\delta_{g,\text{loc}}^{(n)}(\vec{x}, t) = \sum_{\mathcal{O}_m} c_{\mathcal{O}_m}(t) \mathcal{O}_m^{(n)}(\vec{x}, t) , \quad \delta_g^{(n)}(\vec{x}, t) = \sum_{\mathcal{O}_m} \sum_{\alpha=1}^{n-m+1} c_{\mathcal{O}_m, \alpha}(t) \mathbb{C}_{\mathcal{O}_m, \alpha}^{(n)}(\vec{x}, t)$$

- it turns out that up to 4th order in PT, the two basis of operators were identical.
- but at 5th order they are not!
  - out of 29 independent operators, 3 cannot be written as local in time ones.
- $\Rightarrow$  By looking at, eg,

$$\langle \delta_{g_1}^{(5)}(\vec{x}_1) \delta_{g_2}^{(1)}(\vec{x}_2) \delta_{g_3}^{(1)}(\vec{x}_3) \delta_{g_4}^{(1)}(\vec{x}_4) \delta_{g_5}^{(1)}(\vec{x}_5) \delta_{g_6}^{(1)}(\vec{x}_6) \rangle$$

- we can detect these biases, and, from their size, determine:
  - the order of magnitude of the formation time of galaxies
  - direct evidence that the universe lasted 13 Billion years

# Consequences of non-locality in time

- more on time-non-locality:
  - if formation time is fast,  $1/\omega$ , we can Taylor expand the Kernels:

$$c_{\mathcal{O}_m, \alpha}(t) \approx c_{\mathcal{O}_m}(t) \left( 1 + g_{\mathcal{O}_m, \alpha}(t) \frac{H}{\omega} + \dots \right) ,$$

- so these terms would be suppressed, and we could therefore determine a fast formation time.

# Consequences of non-locality in time

- Nice recursion relations for these operators:

$$\bullet \quad [\mathcal{O}_m(\vec{x}_\text{fl}(\vec{x}, t, t'), t')]^{(n)} = \sum_{\alpha=1}^{n-m+1} \left( \frac{D(t')}{D(t)} \right)^{\alpha+m-1} \mathbb{C}_{\mathcal{O}_m, \alpha}^{(n)}(\vec{x}, t)$$

$$\Rightarrow \quad \mathcal{O}_m^{(n)}(\vec{x}, t) = \sum_{\alpha=1}^{n-m+1} \mathbb{C}_{\mathcal{O}_m, \alpha}^{(n)}(\vec{x}, t) ,$$

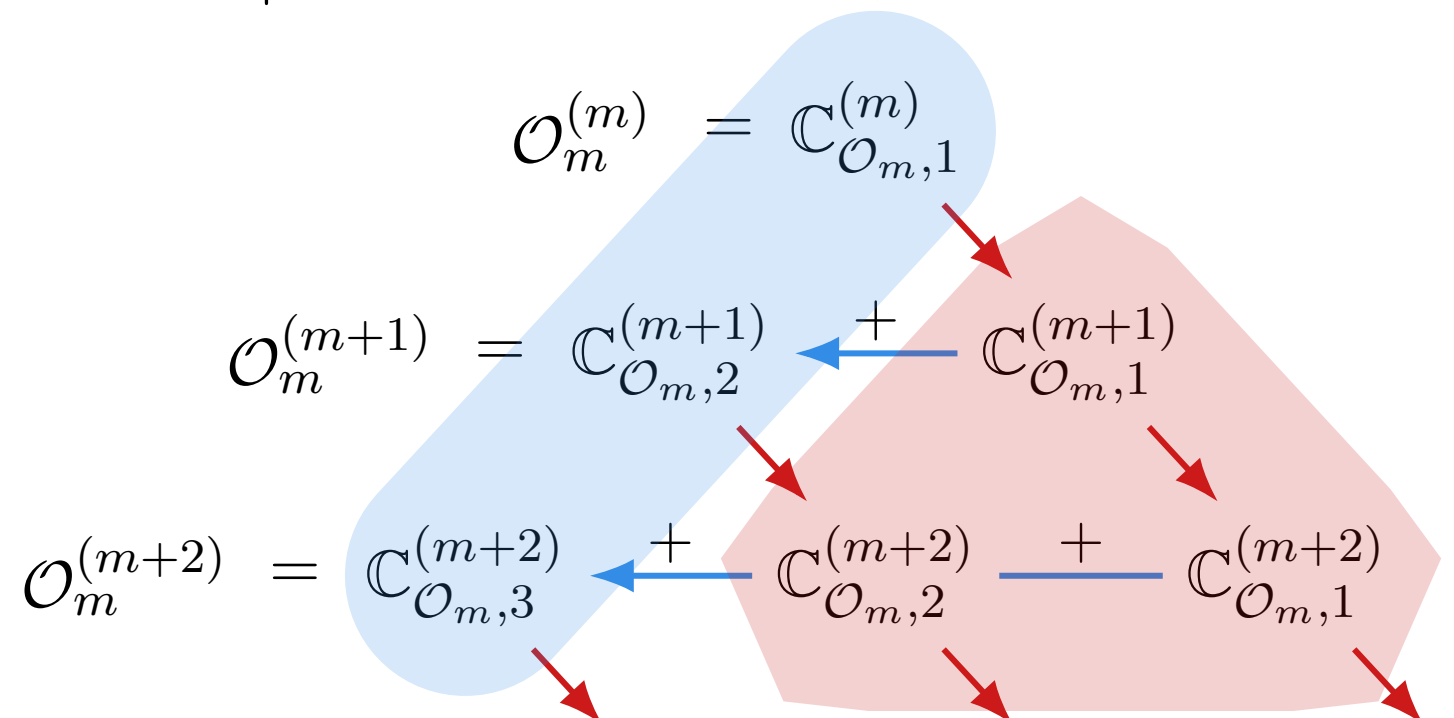
*equal-time completeness relation*

*fluid recursion*

$$\bullet \quad \Rightarrow \quad \mathbb{C}_{\mathcal{O}_m, \alpha}^{(n)}(\vec{x}, t) = \sum_{q=m}^{n-1} \frac{1}{n - \alpha - m + 1} \partial_i \mathbb{C}_{\mathcal{O}_m, \alpha}^{(q)}(\vec{x}, t) \frac{\partial_i}{\partial^2} \theta(\vec{x}, t)^{(n-q)} ,$$

- Easy higher order:

$\Rightarrow$

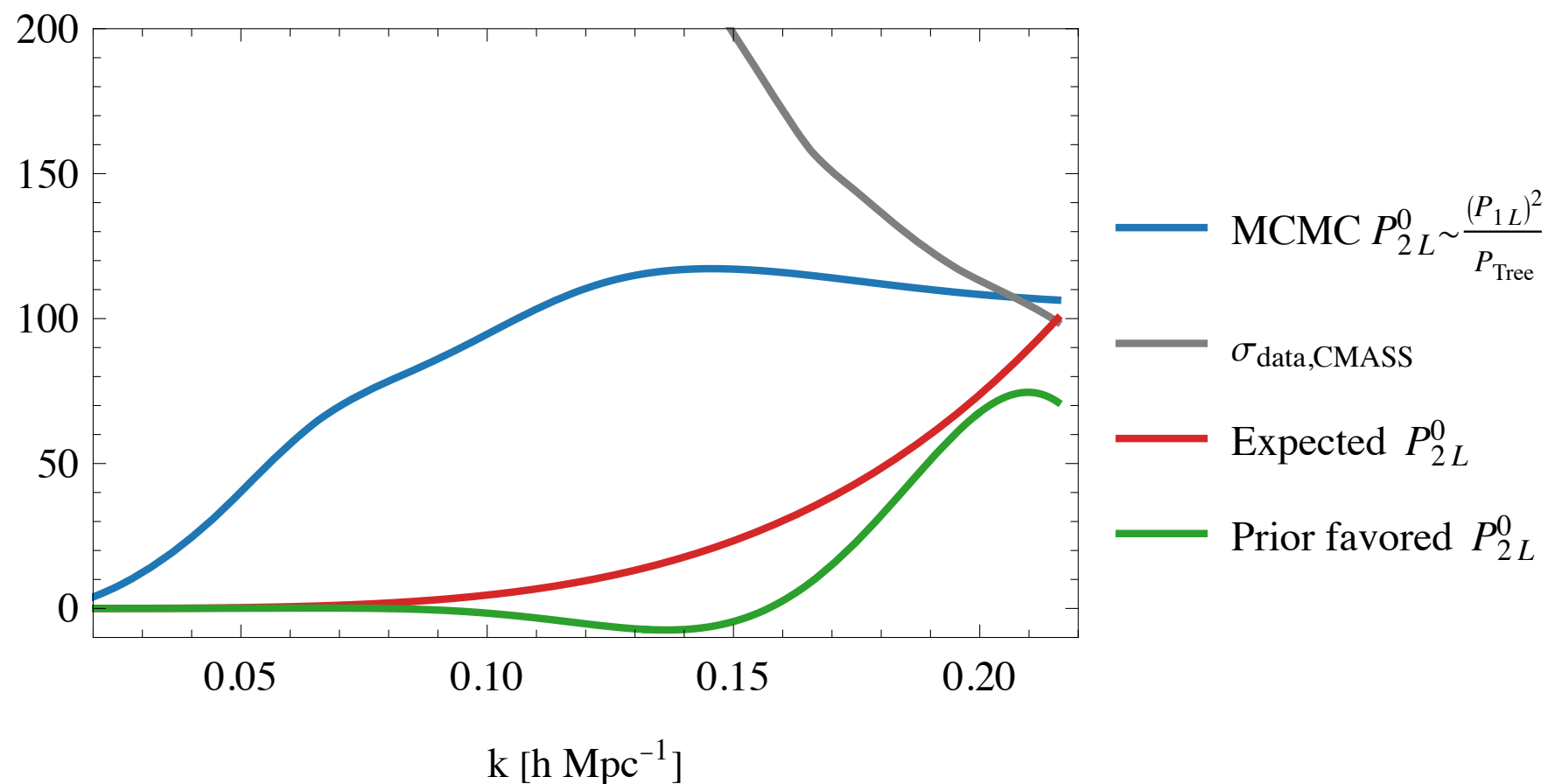


# Peeking into the next Decade

with Donath, Bracanga and Zheng **to appear**

# Next Decade

- After validating our technique against the MCMC's on BOSS data, we Fisher forecast for DESI and Megamapper
- Prediction of one-loop Power Spectrum and Bispectrum
- We introduce a '*perturbativity prior*': impose expected size and scaling of loop



- Also a '*galaxy formation prior*', 0.3 in each EFT-parameter

# Results: Non-Gaussianities

BOSS: $\sigma(\cdot)$	$f_{\text{NL}}^{\text{loc.}}$	$f_{\text{NL}}^{\text{eq.}}$	$f_{\text{NL}}^{\text{orth.}}$
$P+B_{\text{Tree}}$	37	357	142
$P+B$	23	253	67
$P+B+\text{p.p.}$	17	228	62
$P+B+\text{p.p.}+\text{g.p.}$	15	163	49

DESI: $\sigma(\cdot)$	$f_{\text{NL}}^{\text{loc.}}$	$f_{\text{NL}}^{\text{eq.}}$	$f_{\text{NL}}^{\text{orth.}}$
$P+B_{\text{Tree}}$	3.61	142	71.5
$P+B$	3.46	114	30.2
$P+B+\text{p.p.}$	3.26	91.5	27.0
$P+B+\text{p.p.}+\text{g.p.}$	3.19	77.0	21.8

MMo: $\sigma(\cdot)$	$f_{\text{NL}}^{\text{loc.}}$	$f_{\text{NL}}^{\text{eq.}}$	$f_{\text{NL}}^{\text{orth.}}$
$P+B_{\text{Tree}}$	0.29	23.4	8.7
$P+B$	0.27	17.7	4.6
$P+B+\text{p.p.}$	0.26	16.0	4.2
$P+B+\text{p.p.}+\text{g.p.}$	0.26	12.6	3.4

- Just using perturbativity prior, potentially a factor of 20, 3, 6 over Planck!!

# Results: Curvature and Neutrinos

DESI: $\sigma(\cdot)$	$h$	$\ln(10^{10} A_s)$	$\Omega_m$	$n_s$	$\Omega_k$
$P+B$	0.004	0.035	0.002	0.011	0.013
$P+B+\text{p.p.}$	0.004	0.032	0.002	0.008	0.012
$P+B+\text{p.p.}+\text{g.p.}$	0.004	0.025	0.002	0.007	0.009

MMo: $\sigma(\cdot)$	$h$	$\ln(10^{10} A_s)$	$\Omega_m$	$n_s$	$\Omega_k$
$P+B$	0.002	0.0052	0.0003	0.002	0.0015
$P+B+\text{p.p.}$	0.002	0.0046	0.0003	0.002	0.0012
$P+B+\text{p.p.}+\text{g.p.}$	0.002	0.0044	0.0003	0.001	0.0011

- Just using perturbativity prior, potentially factor of 5 over Planck!
  - Important for the landscape of string theory.
- Neutrinos: guaranteed evidence/detection:

$2\sigma$  DESI,       $14\sigma$  MegaMapper

# Where can we make better?

- Shot noise and EFT-parameters:

$\sigma(\cdot)$	$h$	$\ln(10^{10} A_s)$	$\Omega_m$	$n_s$	$f_{\text{NL}}^{\text{loc.}}$	$f_{\text{NL}}^{\text{eq.}}$	$f_{\text{NL}}^{\text{orth.}}$
$P+B$	0.0042	0.020	0.0022	0.010	3.5	114	30
$P+B+g.p. :$	0.0042	0.018	0.0022	0.009	3.4	83	23
$P+B : \text{bias fixed}$	0.0037	0.010	0.0016	0.004	2.0	21	11
$P+B : n_b \rightarrow \infty$	0.0035	0.011	0.0009	0.005	1.7	67	17

DESI

$\sigma(\cdot)$	$h$	$\ln(10^{10} A_s)$	$\Omega_m$	$n_s$	$f_{\text{NL}}^{\text{loc.}}$	$f_{\text{NL}}^{\text{eq.}}$	$f_{\text{NL}}^{\text{orth.}}$
$P+B$	0.0021	0.0047	0.00034	0.0017	0.27	18	4.6
$P+B+g.p. :$	0.0020	0.0045	0.00033	0.016	0.26	13	3.6
$P+B : \text{bias fixed}$	0.0016	0.0034	0.00021	0.0010	0.17	3.6	1.7
$P+B : n_b \rightarrow \infty$	0.00019	0.00045	0.000029	0.00017	0.11	5.4	1.5

MegaMapper



# Summary

- After the initial, successful, application to BOSS data:
  - measurement of cosmological parameters
  - new method to measure Hubble
  - perhaps fixing tension
- the EFTofLSS is starting to look ahead to
  - higher-order and higher-n point functions
  - enlightening what next surveys could do, and how to design them
  - learning about some astrophysics, qualitative facts on the universe