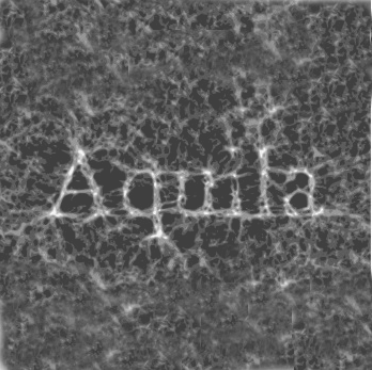


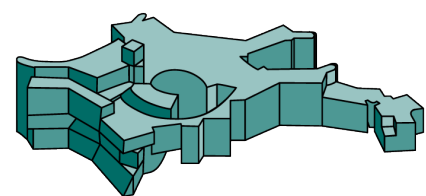


aquila-consortium.org

LEFTfield

$$\mathbf{x} = \mathbf{x}_{\text{fl}}(\tau)$$

EFT meets Field-Level Inference



MAX-PLANCK-INSTITUT
FÜR ASTROPHYSIK



Fabian Schmidt
MPA

with

Ivana Babić, Alex Barreira, Giovanni Cabass, Franz Elsner,
Jens Jasche, Andrija Kostić, Titouan Lazeyras, Guilhem Lavaux,
Minh Nguyen, Julia Stadler, Beatriz Tucci



$$\mathbf{q} = \mathbf{x}_{\text{fl}}(0)$$

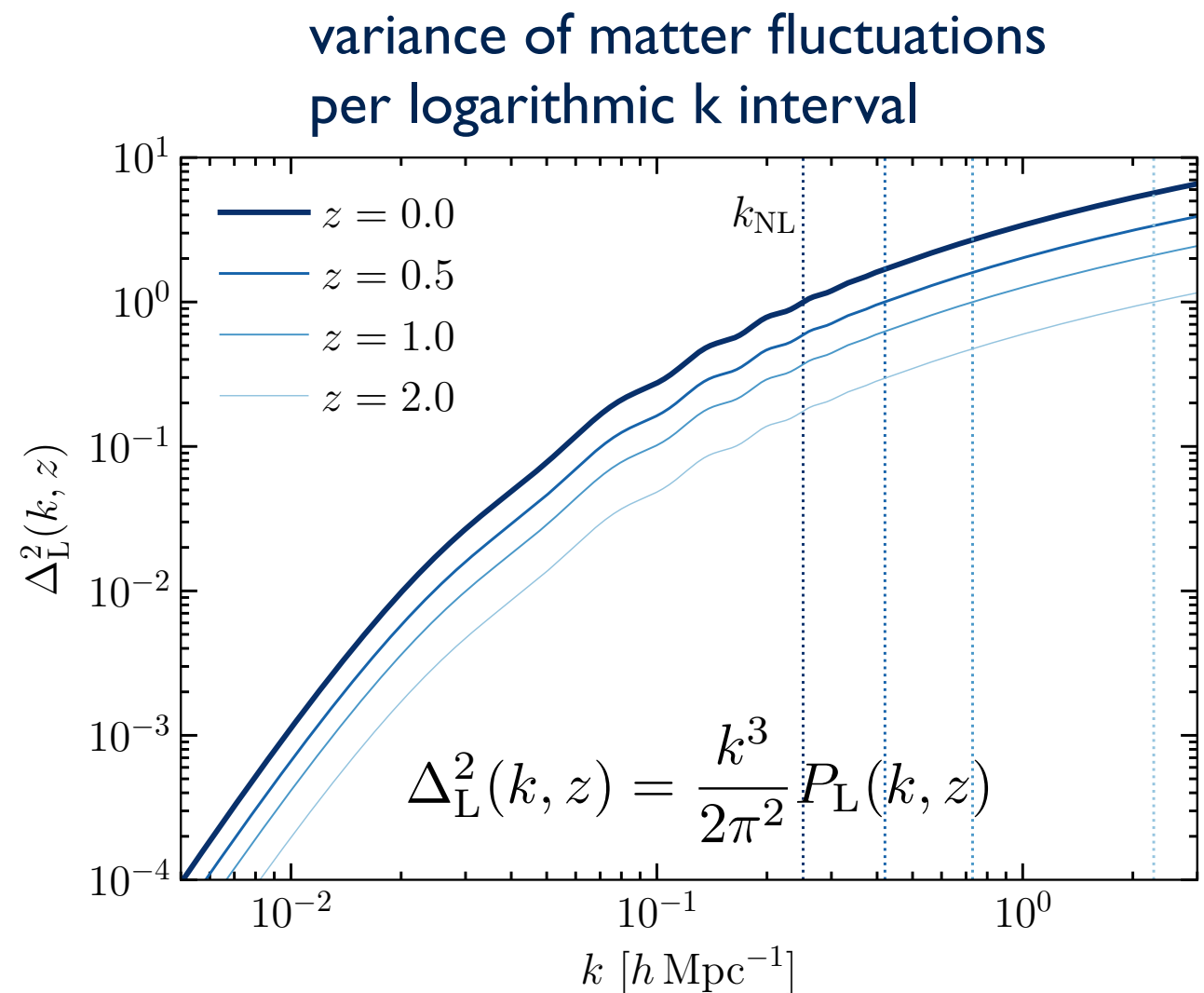
Outline

- A. Theory: EFT at the field level
- B. Inference: Field-level and LFI

A. Theory: EFT at the field level

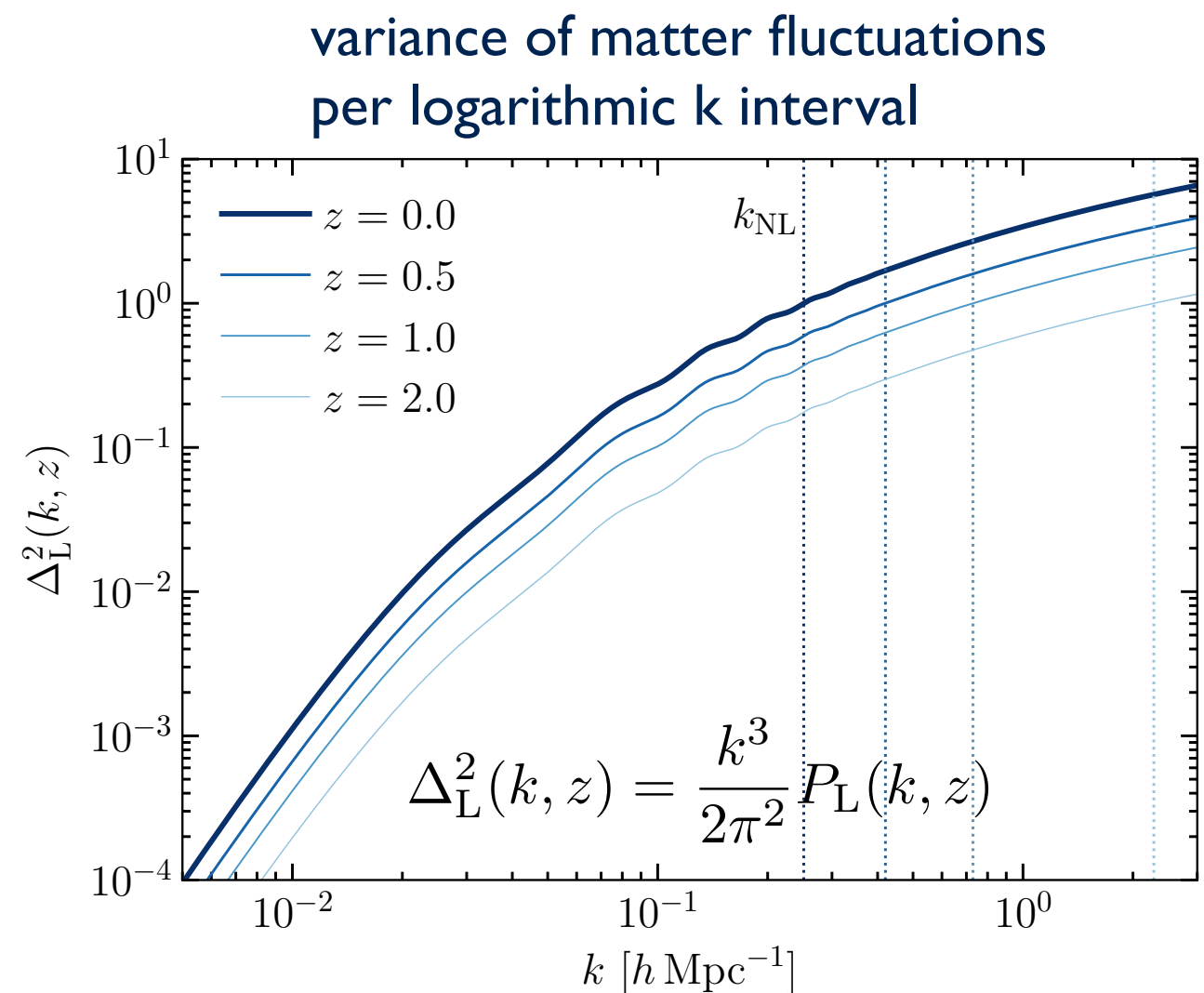
Cold Dark Matter cosmology in a nutshell

- *Inflation+CMB* $\rightarrow$ scale-invariant, adiabatic, approx. Gaussian **initial conditions**
- Large-scale fluctuations are small (still linear today)
- Structure forms *hierarchically from small to large scales*
- *Perturbative expansion* in fluctuations on large scales



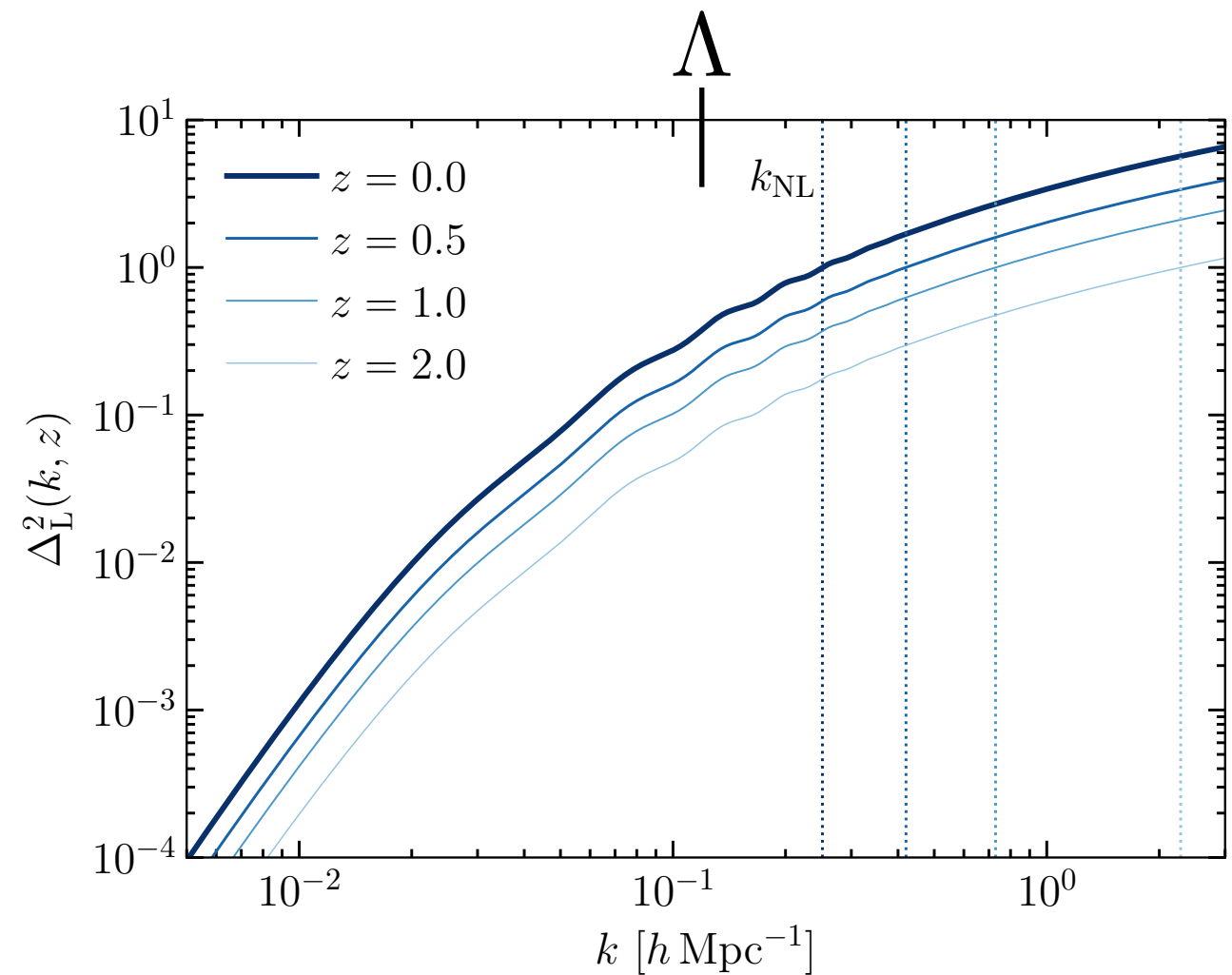
Theory of galaxy clustering

- Perturbations in our universe are small on large scales
 - Perturbation theory works on **quasilinear scales** $k < k_{\text{NL}}$
- Goal: **describe galaxy clustering** up to a given scale and accuracy using a **finite number of free (A) bias parameters** and **(B) stochastic amplitudes**



EFT approach

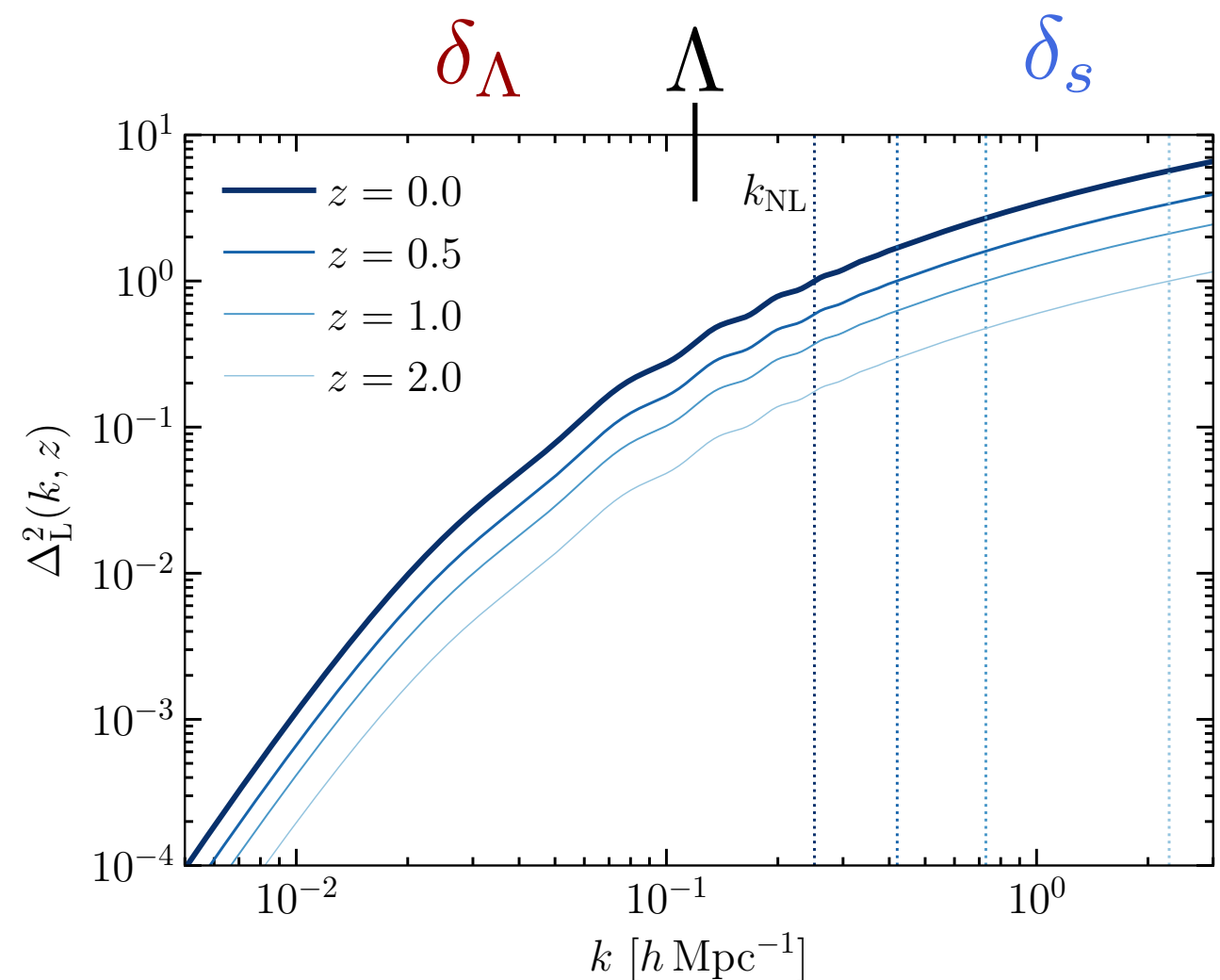
- Idea: trust our theory for $k < \Lambda$



EFT approach

- Idea: trust our theory for $k < \Lambda$
- Split *initial* perturbations into large scale ($< \Lambda$) and small scale ($\geq \Lambda$):

$$\delta(\boldsymbol{x}, \tau) \equiv \frac{\rho_m(\boldsymbol{x}, \tau)}{\bar{\rho}_m(\tau)} - 1 = \delta_\Lambda + \delta_s$$

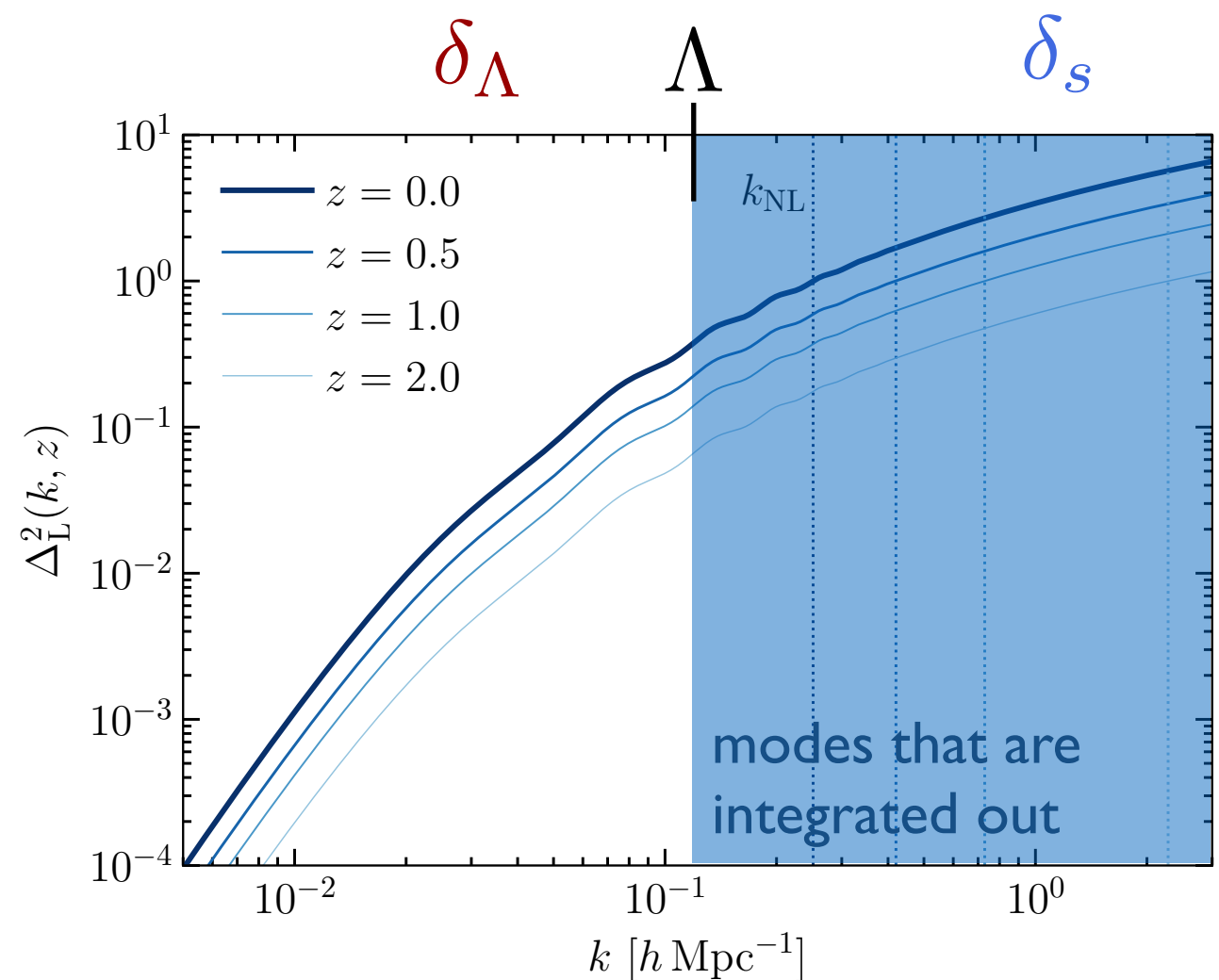


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- Then, we integrate out (marginalize over) perturbations with $k > \Lambda$

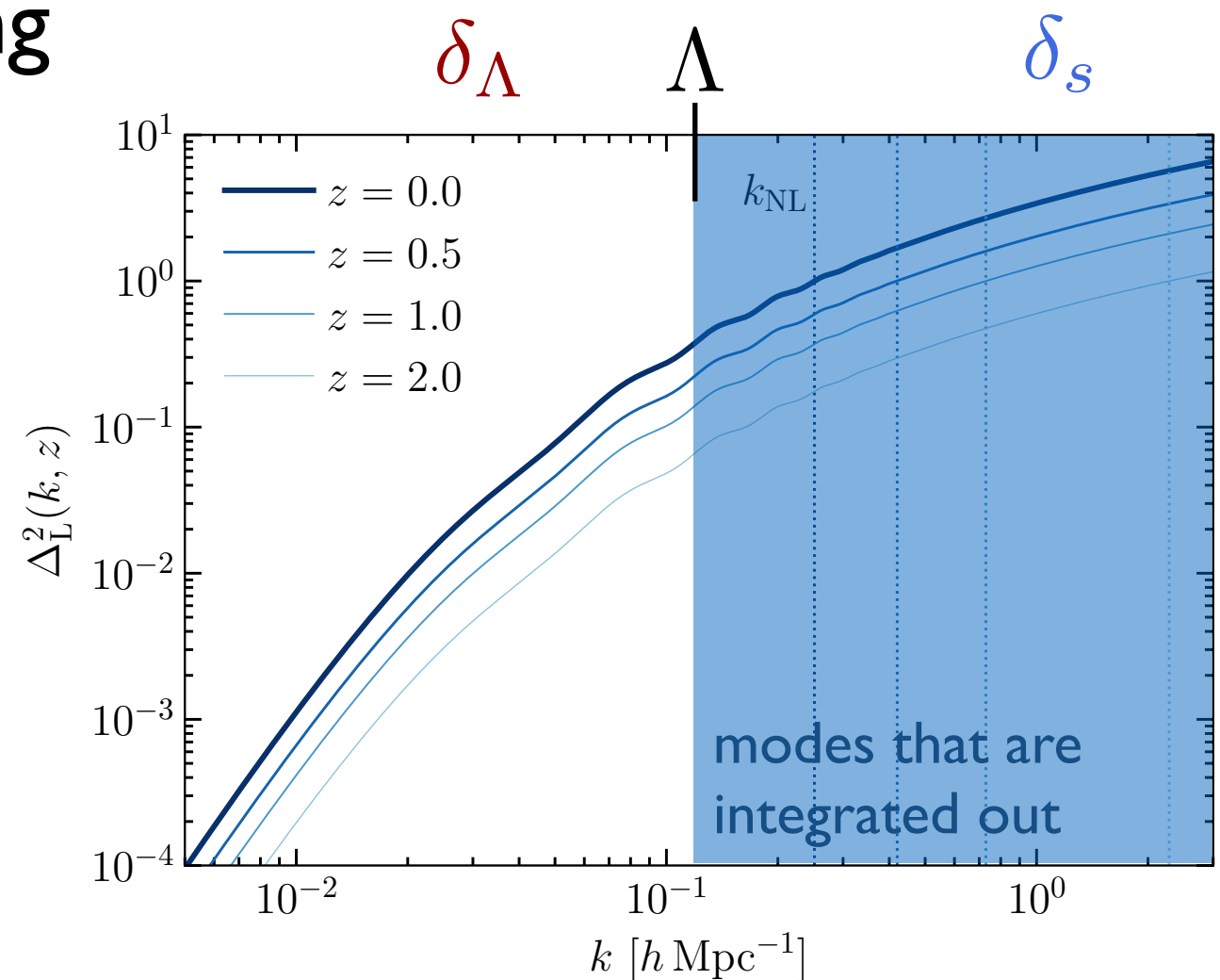


(A) Bias

- Incorporate effect of **large-scale perturbations** explicitly using bias expansion, with free coefficients b_O

$$\delta_g(\boldsymbol{x}) = \sum_O b_O \boldsymbol{O}(\boldsymbol{x})$$

- Fields O are constructed from δ_Λ

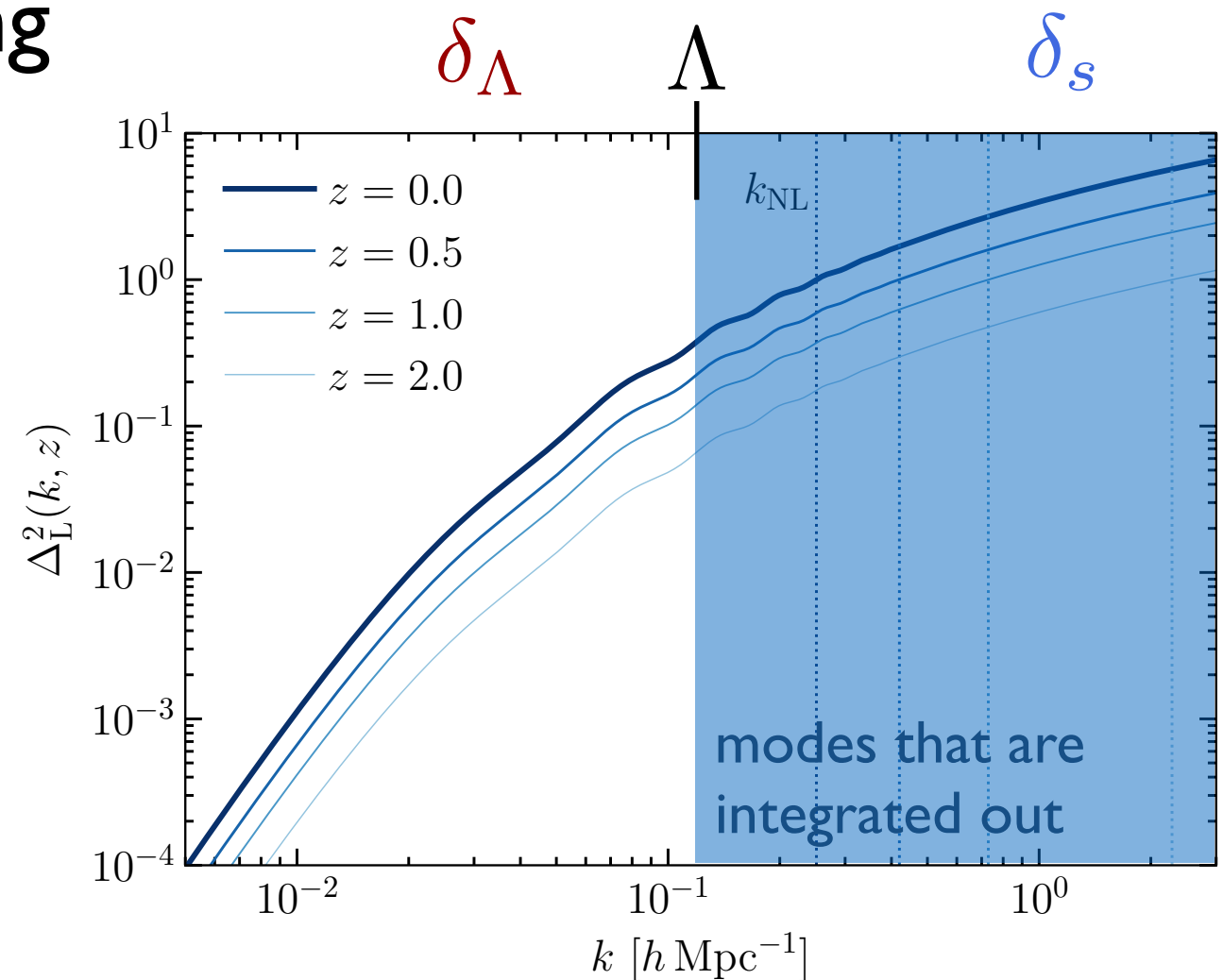


(A) Bias

- Incorporate effect of **large-scale perturbations** explicitly using bias expansion, with free coefficients b_O

$$\delta_g(\boldsymbol{x}) = \sum_O b_O \textcolor{red}{O}(\textcolor{red}{\boldsymbol{x}}) + \textcolor{blue}{\varepsilon}(\textcolor{blue}{\boldsymbol{x}})$$

- Fields O are constructed from δ_Λ
- **Small-scale perturbations** add noise ε

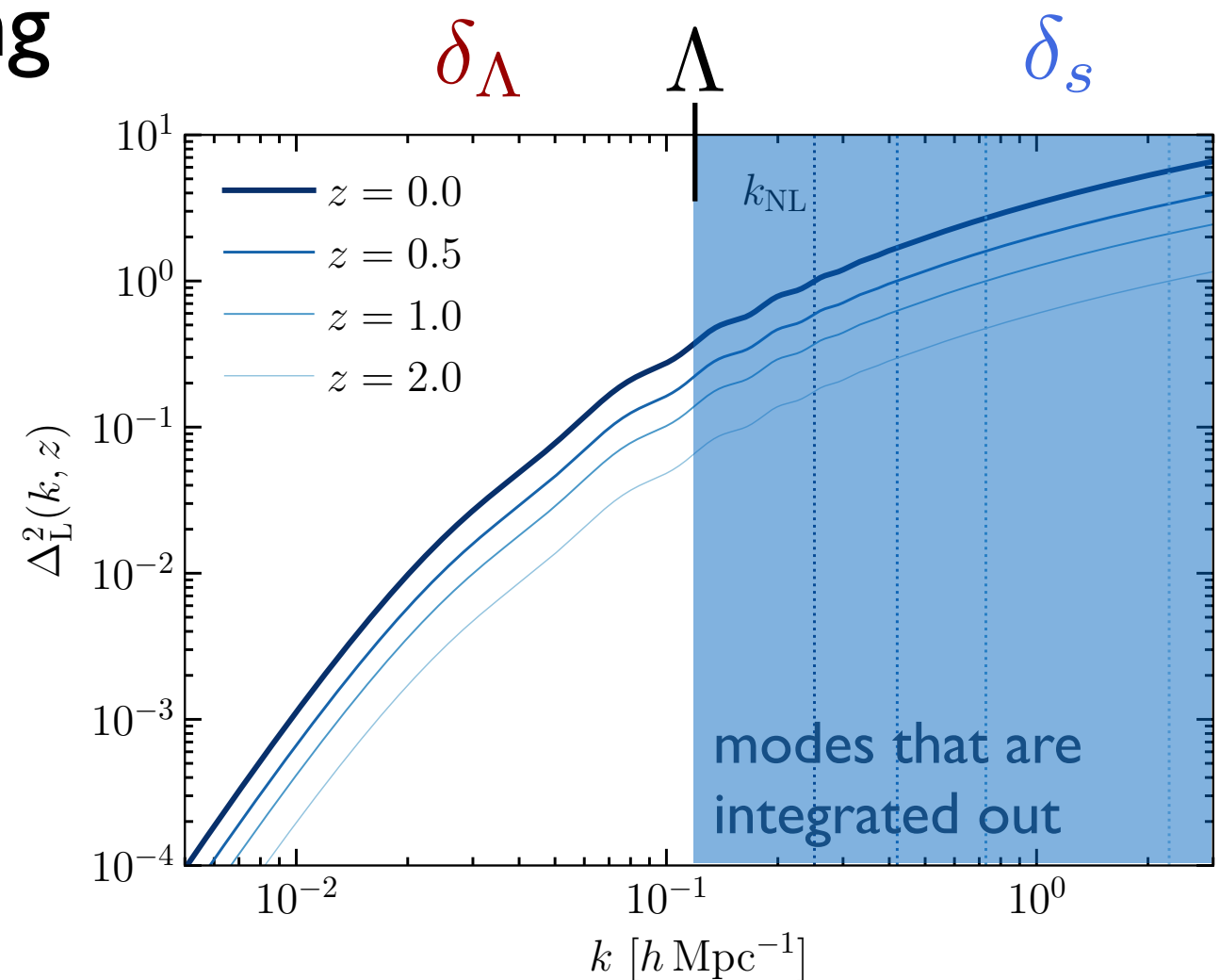


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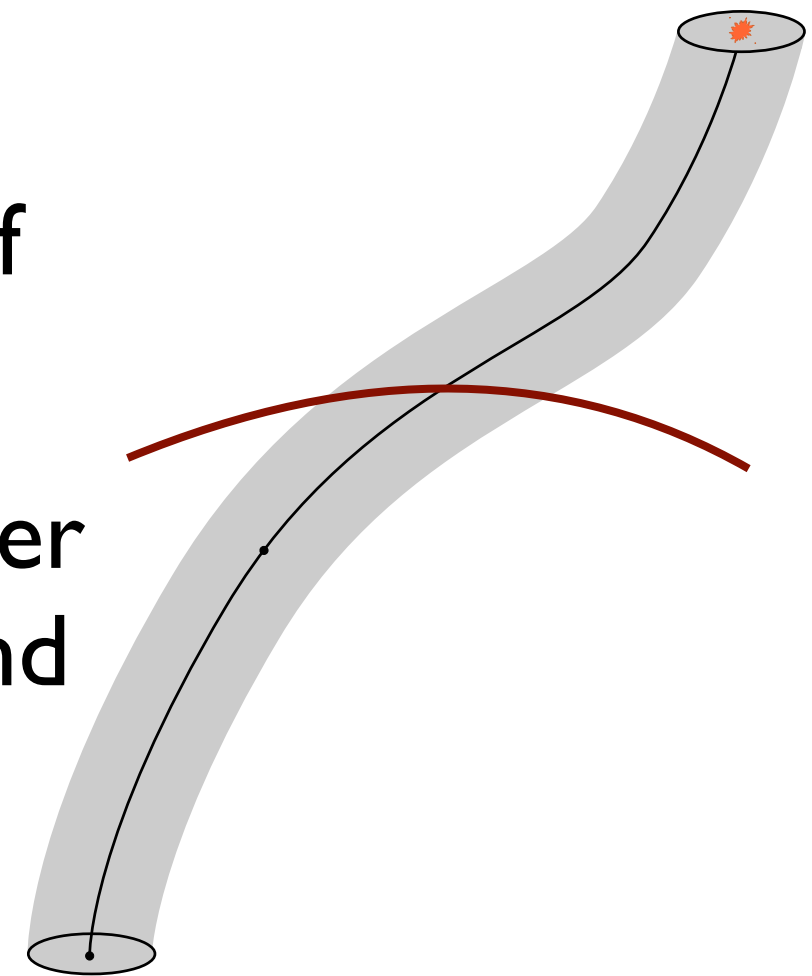
$$\delta_g(\boldsymbol{x}) = \sum_O b_O \boldsymbol{O}(\boldsymbol{x}) + \varepsilon(\boldsymbol{x})$$

- Fields O are constructed from δ_Λ
- Can understand this as expanding the source term in the continuity eqn: $\dot{n}_g(\boldsymbol{x}, t) + \partial_i(n_g u_g^i(\boldsymbol{x}, t)) = f_{\text{eff}}[\partial_i \partial_j \Phi(\boldsymbol{x}, t), \boldsymbol{x}]$



(A) Bias

- Which bias terms $O(\boldsymbol{x})$ we need to include:
 - Well understood by now
 - Include dependence on full history of structure formation
 - Includes “local bias” (powers of matter density) as well as tidal fields, time and space derivatives thereof
- Displacement terms protected by equivalence principle have *fixed* coefficients!



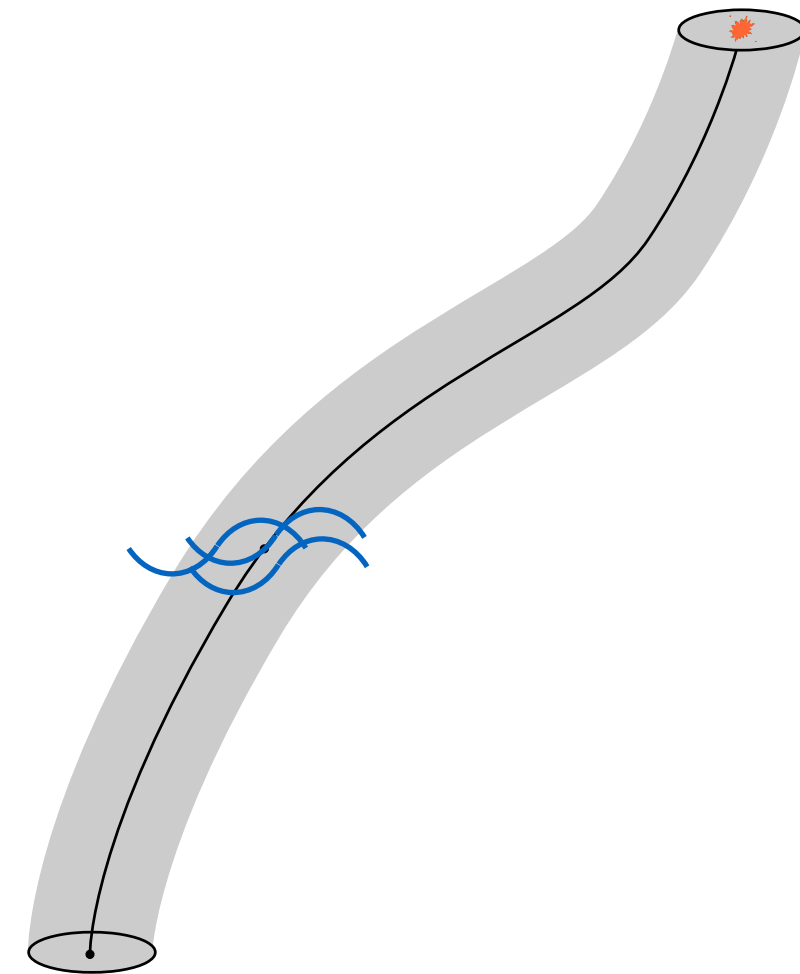
Bias: details

- The bias expansion should contain all possible terms we can make out of time and spatial derivatives of density and tidal field
 - Spatial derivatives suppressed by $k R_*$, where R_* is spatial length scale associated with galaxy formation
- Convenient, complete expansion in terms of PT contributions to Lagrangian distortion tensor $\mathbf{M}^{(n)}$ up to a given order ($n \leq N$):

1 st	$\text{tr}[M^{(1)}] \Leftrightarrow \delta$	$\frac{\partial x^i}{\partial q^j} = \delta^i_j + \partial_{q,j} s^i(\mathbf{q}, \tau)$ $=: \delta^i_j + M^i_j(\mathbf{q}, \tau)$
2 nd	$\text{tr}[(M^{(1)})^2], (\text{tr}[M^{(1)}])^2$	
3 rd	$\text{tr}[(M^{(1)})^3], \text{tr}[(M^{(1)})^2] \text{tr}[M^{(1)}], (\text{tr}[M^{(1)}])^3, \text{tr}[M^{(1)} M^{(2)}]$	
4 th	$\text{tr}[(M^{(1)})^4], \text{tr}[(M^{(1)})^3] \text{tr}[M^{(1)}], \left(\text{tr}[(M^{(1)})^2]\right)^2, (\text{tr}[M^{(1)}])^4,$ $\text{tr}[M^{(1)}] \text{tr}[M^{(1)} M^{(2)}], \text{tr}[M^{(1)} M^{(1)} M^{(2)}], \text{tr}[M^{(1)} M^{(3)}], \text{tr}[M^{(2)} M^{(2)}].$	

(B) Stochasticity

- ε arises from local (in real space) superposition of many small-scale perturbations
- Central limit theorem: $\varepsilon(\mathbf{k})$ is approximately Gaussian distributed (the lower k , the more Gaussian it is)
- Local in real space: power spectrum is white noise at low k , with corrections* $\sim k^2$:



* Also, density dependence: coupling of ε and δ

$$\langle \varepsilon(\mathbf{k}) \varepsilon^*(\mathbf{k}') \rangle = (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') \left[P_\varepsilon + k^2 P_\varepsilon^{\{2\}} + \dots \right]$$

Partition function for galaxies

- Idea: couple solutions to EOM to source term
- After integrating out *initial* modes above Λ ,

obtain

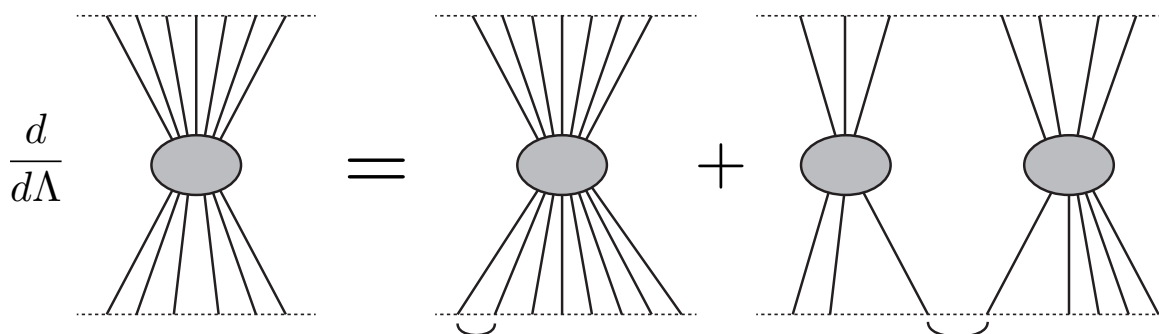
$$\begin{aligned} Z_\Lambda[J_\Lambda] = \int \mathcal{D}\delta_{\text{in},\Lambda} \exp & \left[-\frac{1}{2} \delta_{\text{in},\Lambda}^i \left(P_{\text{L},\Lambda}^{-1} \right)_{ij} \delta_{\text{in},\Lambda}^j \right. \\ & + J_{\Lambda,i} \sum_{m=1}^{\infty} \frac{1}{m!} K_{i_1 \dots i_m}^i(\Lambda) \delta_{\text{in},\Lambda}^{i_1} \dots \delta_{\text{in},\Lambda}^{i_m} \\ & + \frac{1}{2} J_{\Lambda,i} J_{\Lambda,j} \sum_{m=0}^{\infty} \frac{1}{m!} K_{i_1 \dots i_m}^{ij}(\Lambda) \delta_{\text{in},\Lambda}^{i_1} \dots \delta_{\text{in},\Lambda}^{i_m} \\ & \left. + \frac{1}{6} J_{\Lambda,i} J_{\Lambda,j} J_{\Lambda,k} \sum_{m=0}^{\infty} \frac{1}{m!} K_{i_1 \dots i_m}^{ijk}(\Lambda) \delta_{\text{in},\Lambda}^{i_1} \dots \delta_{\text{in},\Lambda}^{i_m} + \dots \right]. \end{aligned}$$

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- Doesn't depend on cutoff Λ $\rightarrow$ RG equations



$P_{\text{L},\Lambda}$ is cut linear propagator

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- $O(J)$ contributions: bias

$$\delta_{g,\text{det},\Lambda}^i[\delta_{\text{in},\Lambda}] = \sum_{m=1}^{\infty} \frac{1}{m!} K_{i_1 \dots i_m}^i(\Lambda) \delta_{\text{in},\Lambda}^{i_1} \dots \delta_{\text{in},\Lambda}^{i_m} \quad ; \quad \frac{1}{m!} K_{i_1 \dots i_m}^i(\Lambda) = \sum_{n=1}^m \sum_{O[n]} b_O(\Lambda) F_m^O(\mathbf{k}_{i_1}, \dots, \mathbf{k}_{i_m})$$

- $O(J^2), O(J^3), \dots$ contributions: stochasticity

B. Inference: Field-level and LFI

A broad view of cosmology inference

- Given **cosmological parameters θ** , we can hope to predict
 1. Statistics of initial conditions
 2. How a given $\delta_{\text{in}}(x)$ evolves into the final density field

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A broad view of cosmology inference

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1. Statistics of initial conditions **Prior** $P_{\text{prior}}(\vec{\delta}_{\text{in}}, \theta)$
2. How a given $\delta_{\text{in}}(x)$ evolves into the final density field **deterministic evolution**
 $\vec{\delta}_{\text{fwd}}[\vec{\delta}_{\text{in}}, \theta]$

Bayesian cosmology inference

- The *full posterior of cosmological parameters given the data* is then given by

$$P(\theta) \propto \int \mathcal{D}\delta_{\text{in}} P\left(\delta_g \middle| \delta_{\text{fwd}}[\delta_{\text{in}}, \theta]\right) P_{\text{prior}}(\delta_{\text{in}}, \theta)$$

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Multivariate Gaussian, diagonal
covariance in Fourier space



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conditional probability of galaxy density given matter density
- contains all physics of galaxy formation

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Functional integral

Bayesian cosmology inference

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- Standard approach proceeds via *data compression*: replace galaxy density field with much smaller data vector (e.g., power spectrum in bins of k)

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- Standard approach proceeds via *data compression*: replace galaxy density field with much smaller data vector (e.g., power spectrum in bins of k)
- Then, the functional integral over initial conditions (a.k.a taking ensemble average) is done either
 - semi-analytically (loop integrals in PT / EFT approach - formally, sending Λ to infinity)
 - numerically (emulators based on ensemble of simulations)

Bayesian cosmology inference

$$P(\theta) \propto \int \mathcal{D}\delta_{\text{in}} P\left(\delta_g \middle| \delta_{\text{fwd}}[\delta_{\text{in}}, \theta]\right) P_{\text{prior}}(\delta_{\text{in}}, \theta)$$

- Can we make progress *without* this (lossy) data compression?

Inference beyond the power spectrum

$$P(\theta) \propto \int \mathcal{D}\delta_{\text{in}} P\left(\delta_g \middle| \delta_{\text{fwd}}[\delta_{\text{in}}, \theta]\right) P_{\text{prior}}(\delta_{\text{in}}, \theta)$$

- Yes - basically by doing a Markov Chain Monte Carlo:

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 - Discretize field on grid/lattice (Nyquist frequency = cutoff Λ)
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- Lots of interest in this approach recently

Kitaura & Ensslin, Jasche & Wandelt, Wang, Mo et al, Seljak et al, Jasche & Lavaux (2017), ...

The galaxy likelihood

- Putting numerical challenges aside, we need an expression for the *field-level galaxy likelihood*:
- conditional probability of galaxy density given matter density

$$\begin{aligned} P(\theta) &\propto \int \mathcal{D}\delta_{\text{in}} \, P\left(\delta_g \middle| \delta_{\text{fwd}}[\delta_{\text{in}}, \theta]\right) P_{\text{prior}}(\delta_{\text{in}}, \theta) \\ &= \int d\{b_O\} \, P\left(\delta_g \middle| \delta_{\text{fwd}}[\delta_{\text{in}}, \theta]\right) P_{\text{prior}}(\{b_O\}) \end{aligned}$$

EFT likelihood

- Combining knowledge about **bias** and **stochasticity**, we can write:

$$\delta_g(\mathbf{k}) = \delta_{g,\text{det}}(\mathbf{k}) + \varepsilon(\mathbf{k})$$

$$\delta_{g,\text{det}}(\mathbf{k}) = \sum_O b_O O(\mathbf{k})$$

- and insert $\varepsilon = \delta_g - \delta_{g,\text{det}}$ into the Gaussian PDF for ε :

$$P[\varepsilon] \propto \exp \left[-\frac{1}{2} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{|\varepsilon(\mathbf{k})|^2}{P_\varepsilon(k)} \right]$$

EFT likelihood from the partition function

- Goal: obtain joint likelihood for galaxy density modes below cutoff, $\{\delta_g(\mathbf{k})\}_{|\mathbf{k}| < \Lambda}$.
- Given by functional Fourier transform of partition function:

$$\begin{aligned}\mathcal{P}_\Lambda[\hat{\delta}_{g,\Lambda}] &= \left\langle \delta_D^{[\infty]}(\delta_{g,\Lambda} - \hat{\delta}_{g,\Lambda}) \right\rangle \\ &= \int \mathcal{D}X_\Lambda \left\langle \exp \left[iX_\Lambda^i (\delta_{g,\Lambda} - \hat{\delta}_{g,\Lambda})_i \right] \right\rangle \\ &= \int \mathcal{D}X_\Lambda \exp \left[-iX_\Lambda^i (\hat{\delta}_{g,\Lambda})_i \right] \left\langle \exp \left[iX_\Lambda^i (\delta_{g,\Lambda})_i \right] \right\rangle \\ &= (Z_\Lambda[0])^{-1} \int \mathcal{D}X_\Lambda \exp \left[-iX_\Lambda^i (\hat{\delta}_{g,\Lambda})_i \right] Z_\Lambda[iX_\Lambda].\end{aligned}$$

EFT likelihood

- We obtain the desired conditional probability for δ_g in *Fourier space*:

$$P\left(\vec{\delta}_g \middle| \vec{\delta}\right) \propto \left(\prod_{\mathbf{k} \neq 0}^{\Lambda} \sigma^2(k)\right)^{-1/2} \exp \left[-\frac{1}{2} \sum_{\mathbf{k} \neq 0}^{\Lambda} \frac{1}{\sigma^2(k)} \left| \delta_g(\mathbf{k}) - \delta_{g,\text{det}}(\mathbf{k}) \right|^2 \right]$$

↑ Finite volume in actual data
-> discrete Fourier representation

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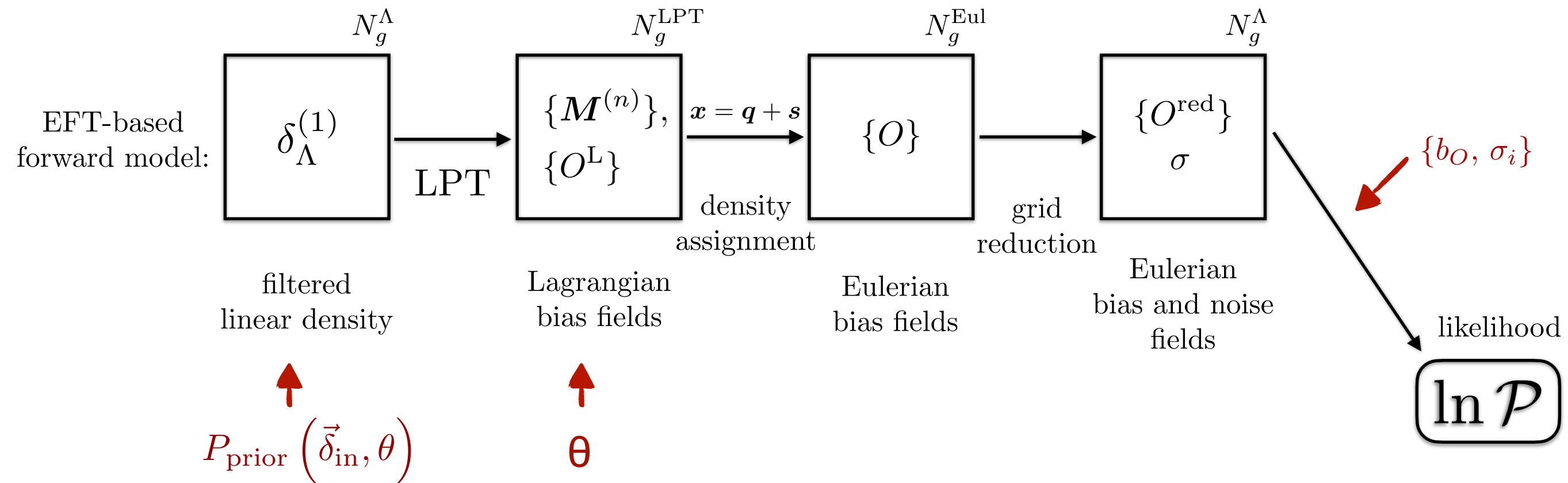
with

$$\delta_{g,\text{det}}(\mathbf{k}) = \sum_O b_O O(\mathbf{k})$$

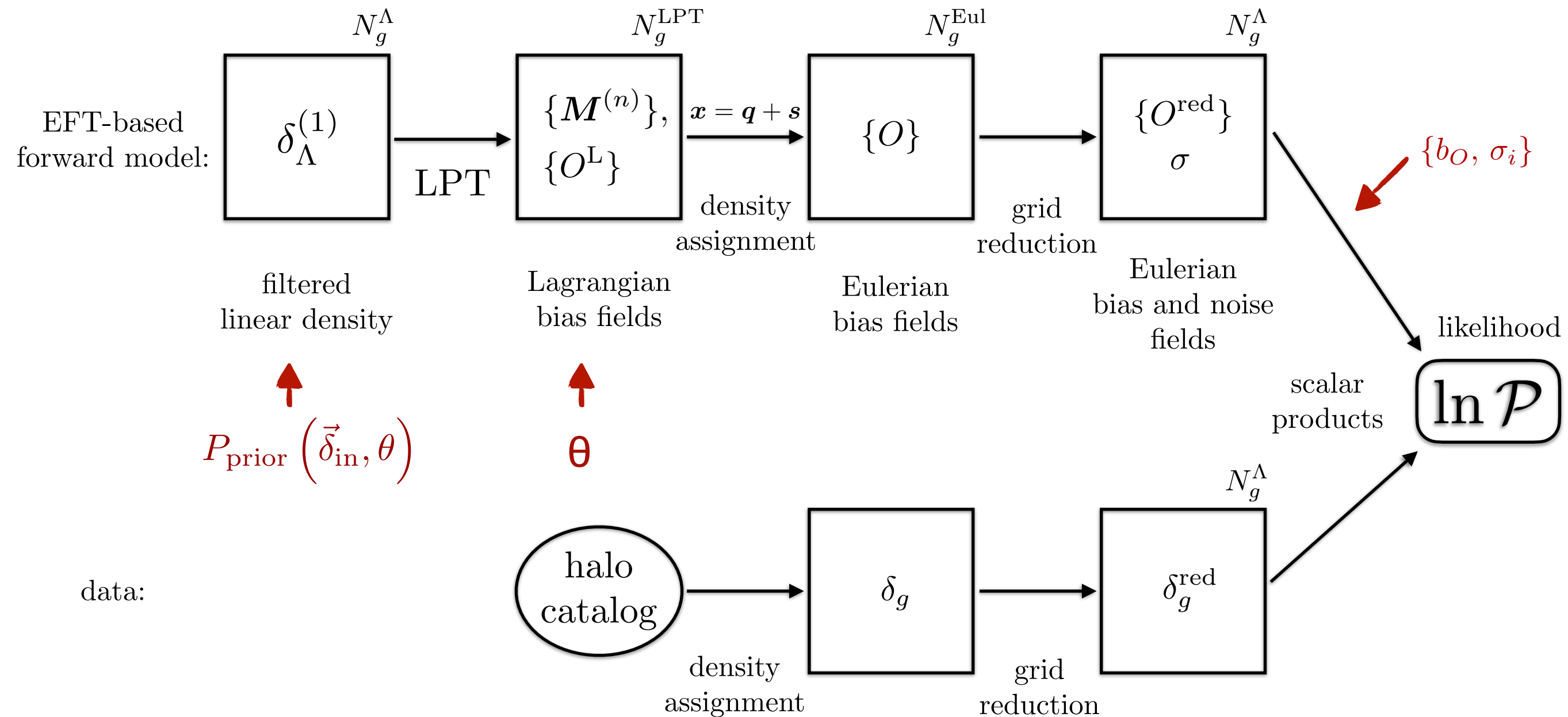
$$\sigma^2(k) = \sigma_0^2 + \sigma_2^2 k^2$$

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Flowchart of the inference pipeline

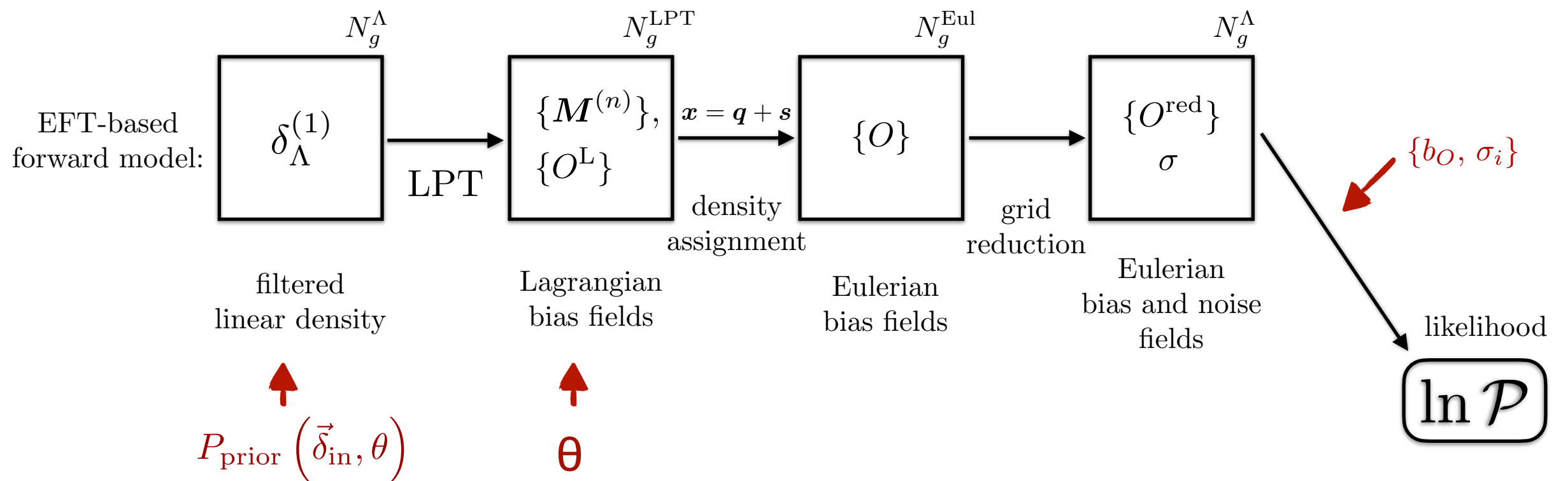


Flowchart of the inference pipeline



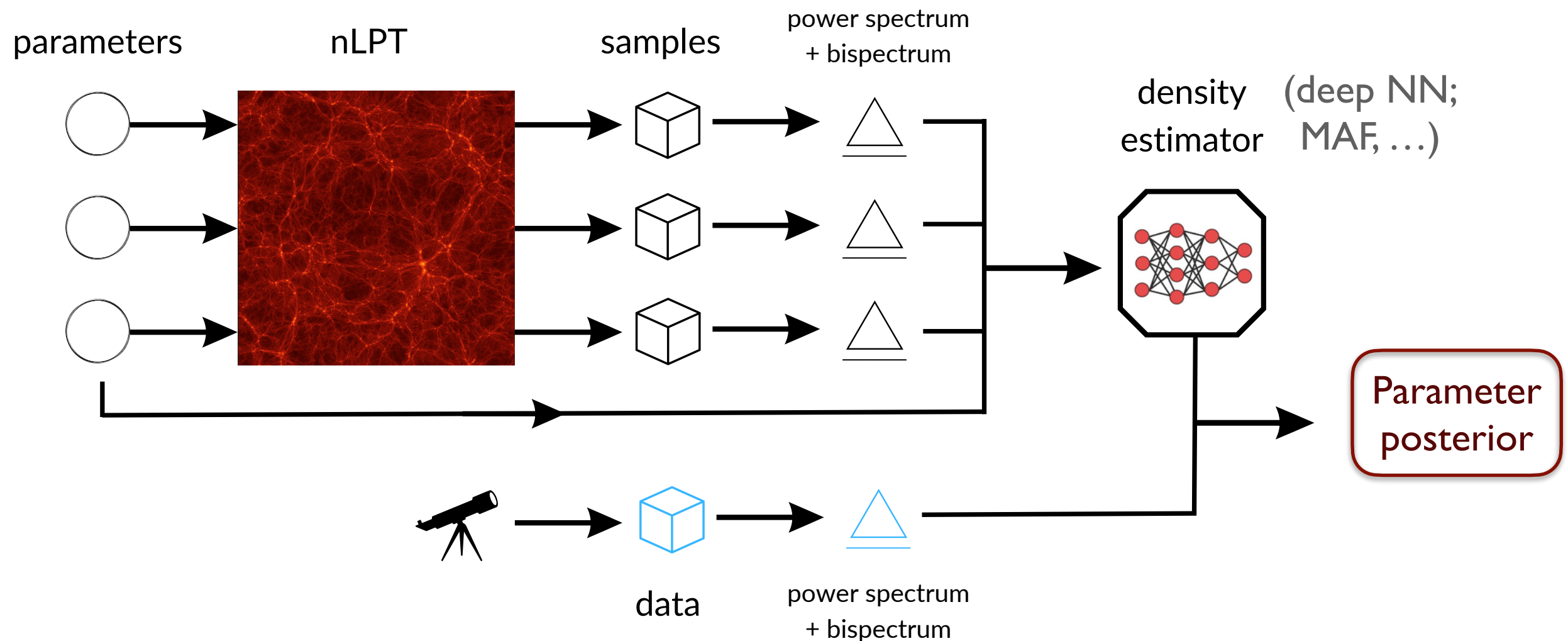
Interlude: implicit-likelihood inference (IFI/LFI)

- Use forward model as *generator* of simulated (“mock”) data:



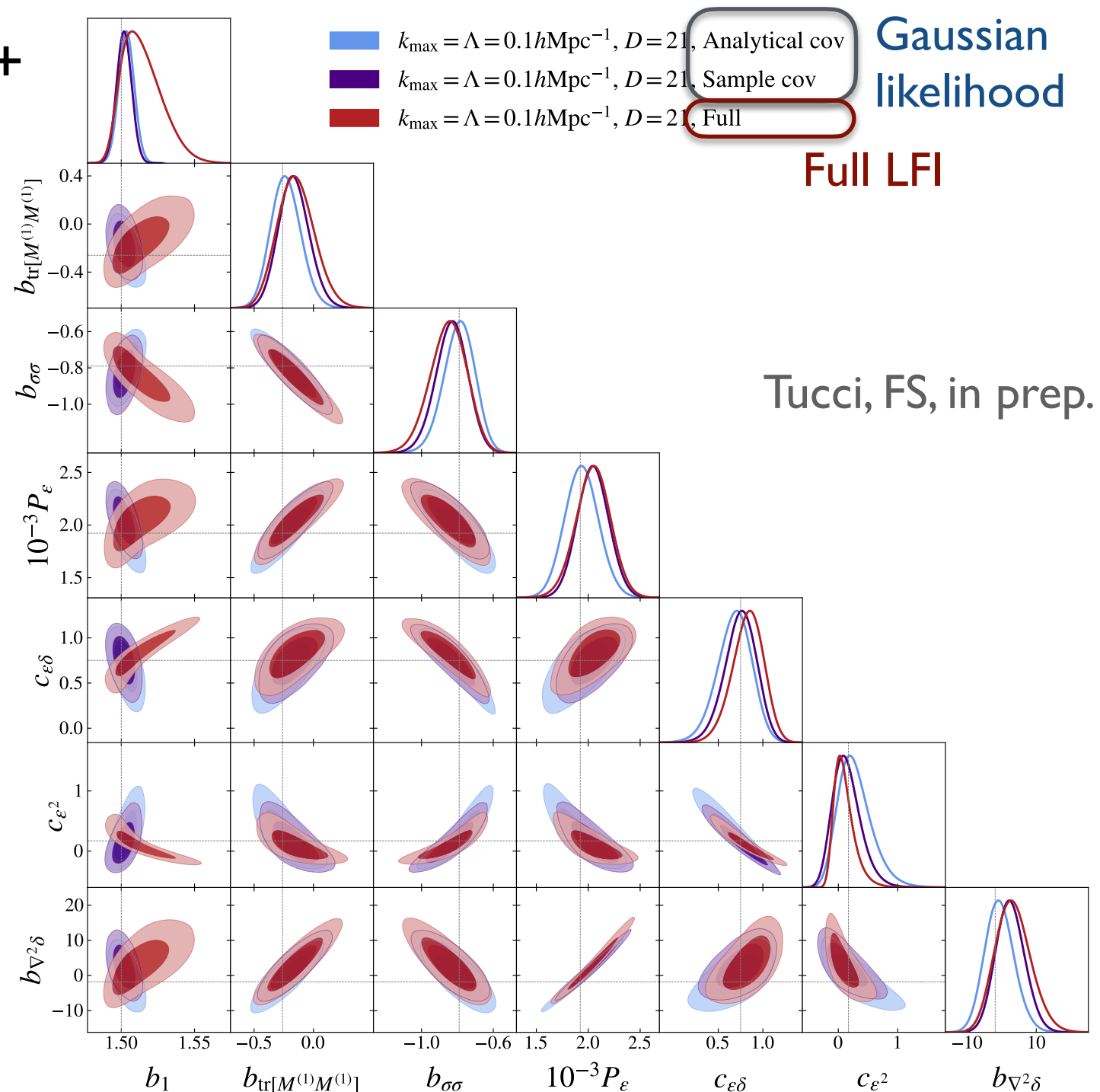
Interlude: implicit-likelihood inference (IFI/LFI)

Beatriz Tucci | PhD Student



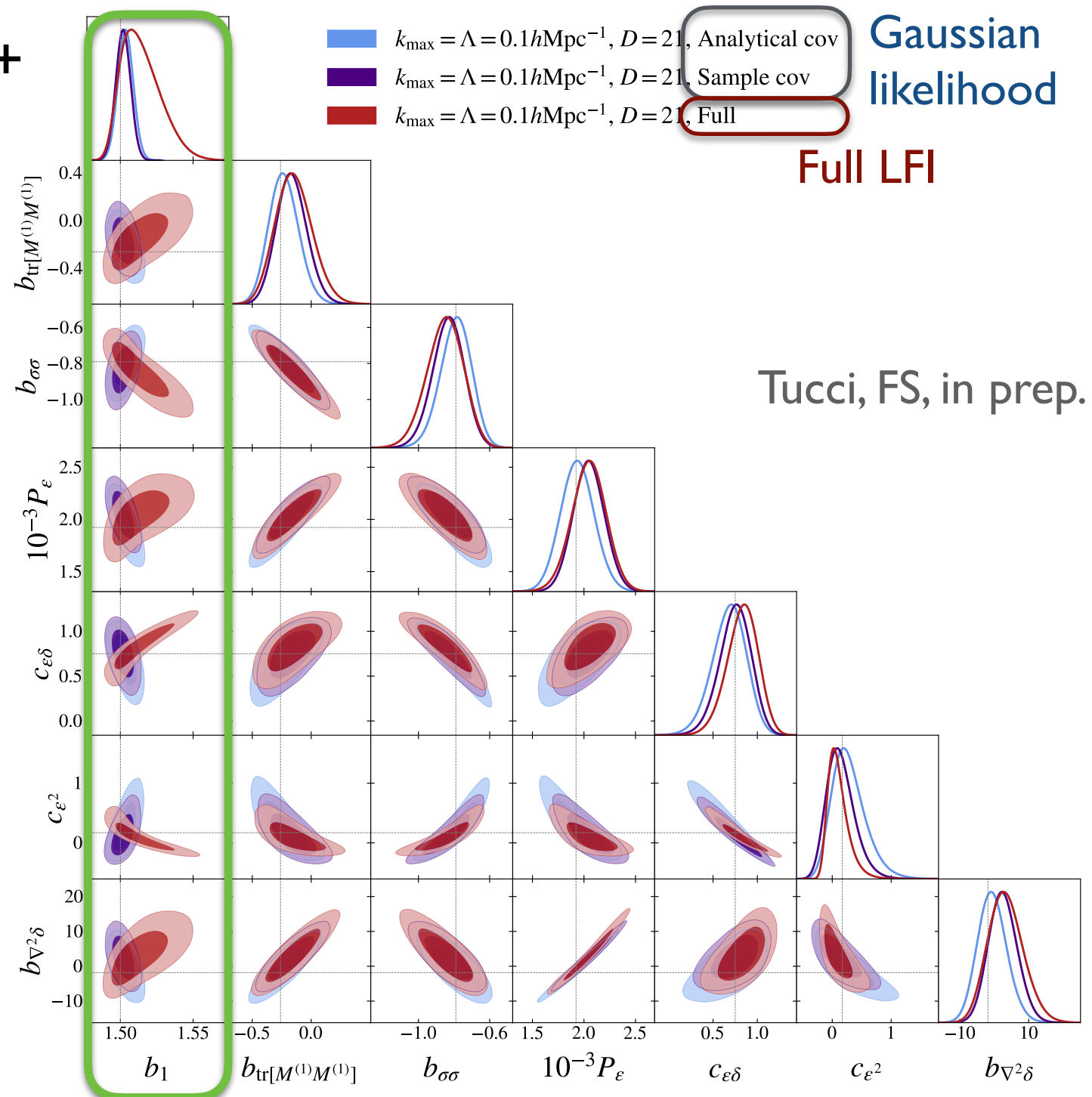
Interlude: implicit-likelihood inference (IFI/LFI)

- Data vector: power spectrum + bispectrum (all configurations)
- Test on mock data set generated from same EFT model
- Fixed cosmology
- Compare **full LFI** with **Gaussian likelihood** of data



Interlude: implicit-likelihood inference (IFI/LFI)

- Data vector: power spectrum + bispectrum (all configurations)
- Test on mock data set generated from same EFT model
- Fixed cosmology
- Compare **full LFI** with **Gaussian likelihood** of data vector
- Gaussian likelihood underestimates error on b_1 by 30-40% in this particular case



Tests at *fixed* initial conditions

- Let's begin with a thought experiment:
 - We are given a halo catalog and the *normalized amplitudes of the initial conditions* for the matter density in the same volume $\delta_{\Lambda}^{(1)}$
 - Can we infer the cosmological parameters from this halo catalog?

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 - We are given a halo catalog and the *normalized amplitudes of the initial conditions* for the matter density in the same volume $\delta_{\Lambda}^{(1)}$
 - Can we infer the cosmological parameters from this halo catalog?
- Near optimal case: no cosmic variance
- Of course, not a real-world example, but applicable to halos (or galaxies) in simulations

Tests at *fixed* initial conditions

- Specifically, can we recover unbiased $\mathcal{A}_s$ (σ_8) from a halo catalog (treating bias parameters as unknown) ?
- Perfect degeneracy between b_1 and σ_8 at linear order; nonlinear information essential

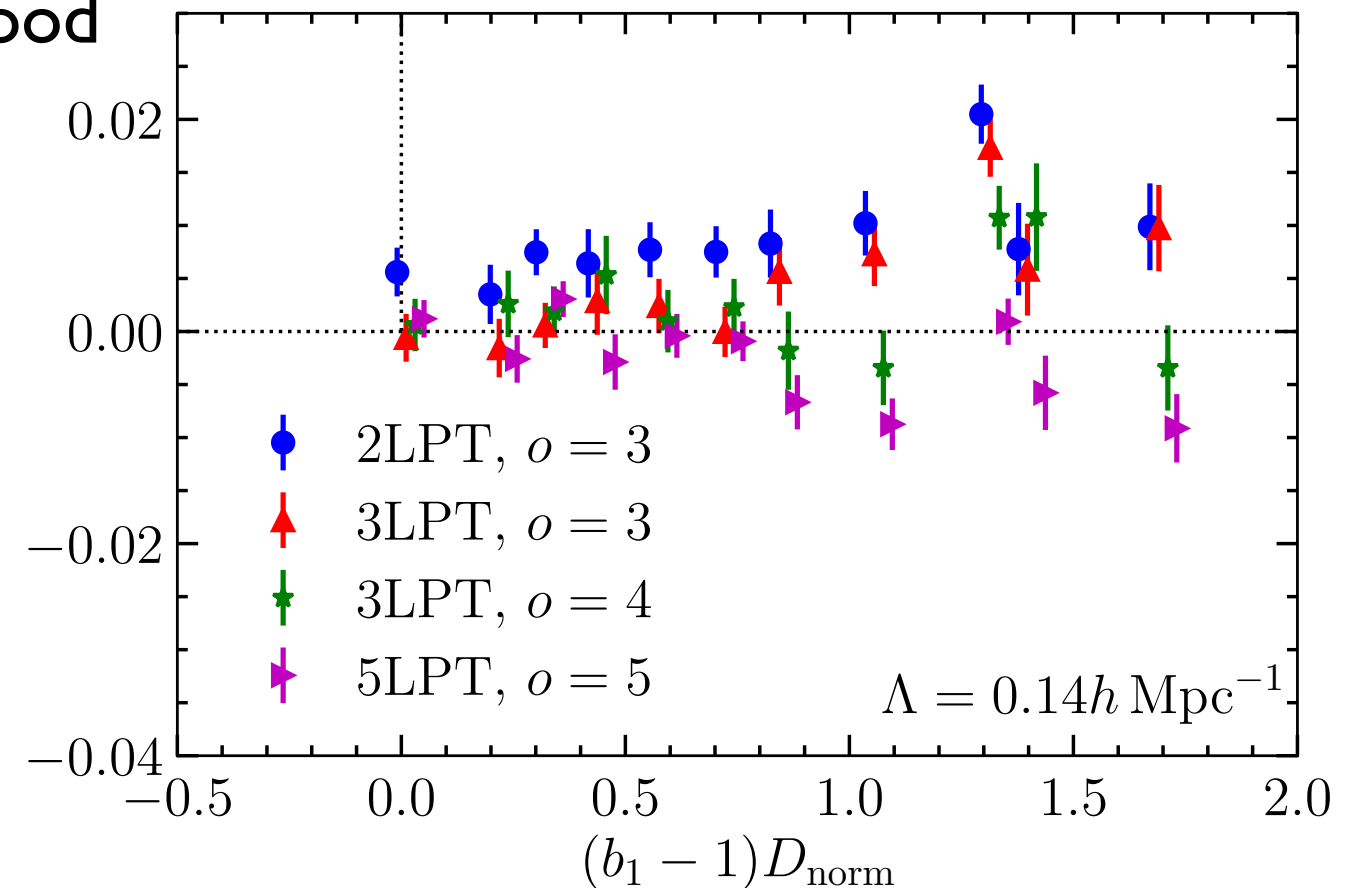
$\mathcal{A}_s$ inference from dark matter halos in simulations

Relative deviation of maximum-likelihood value of σ_8 from ground truth, for different perturbative orders

$L_{\text{box}} = 2000 \text{ Mpc}/h$

$$A_{\text{in}} \equiv \frac{\sigma_8}{\sigma_8^{\text{fid}}}$$

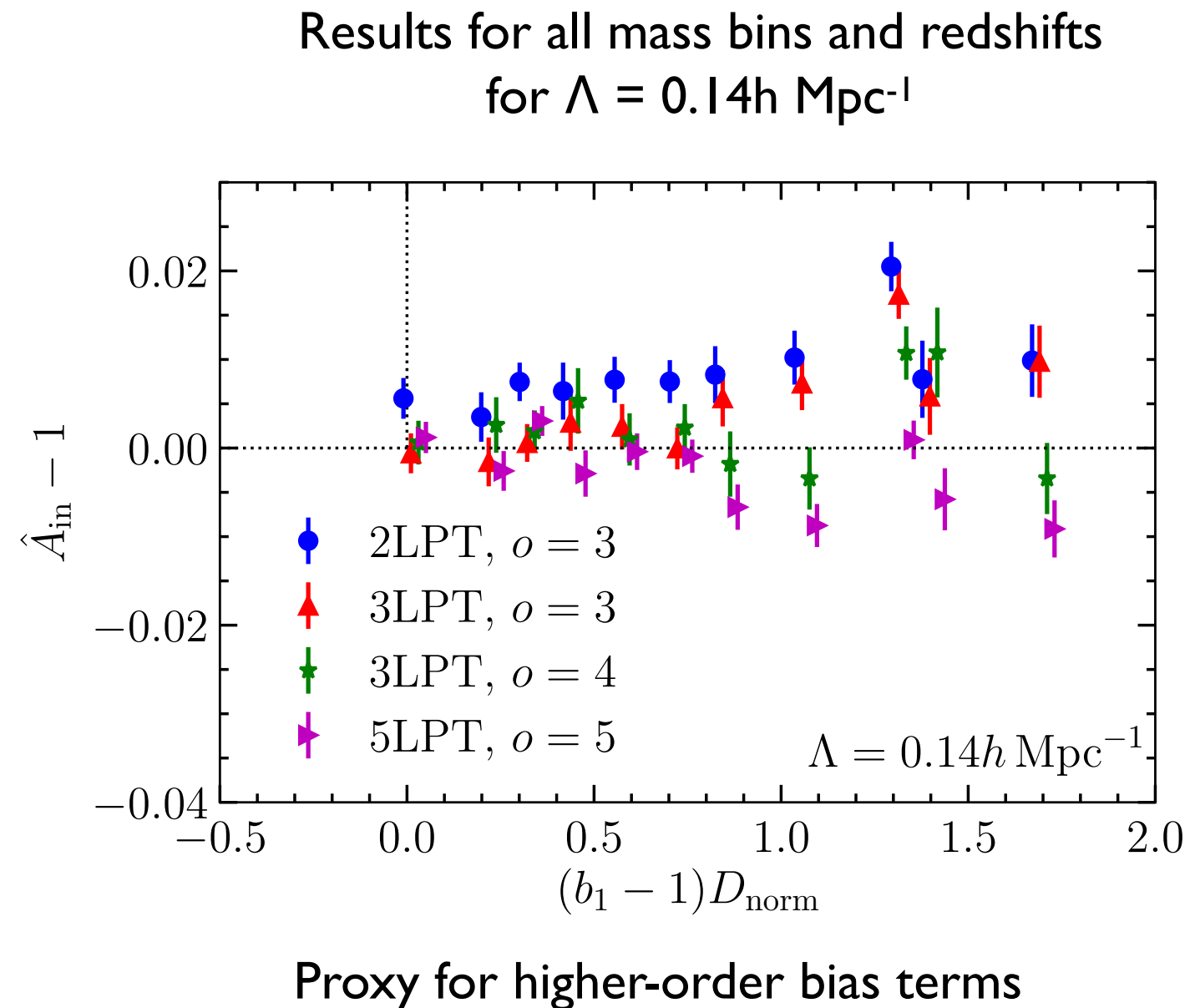
Results for all mass bins and redshifts for $\Lambda = 0.14 h \text{ Mpc}^{-1}$



Proxy for higher-order bias terms

$\mathcal{A}_s$ inference from dark matter halos in simulations

- Residual error in σ_8 at $k < 0.14h/\text{Mpc}$ is $< \sim 1\text{-}2\%$ depending on halos mass and redshift
- Most likely due to higher-order bias, and numerical errors of simulations (transients)

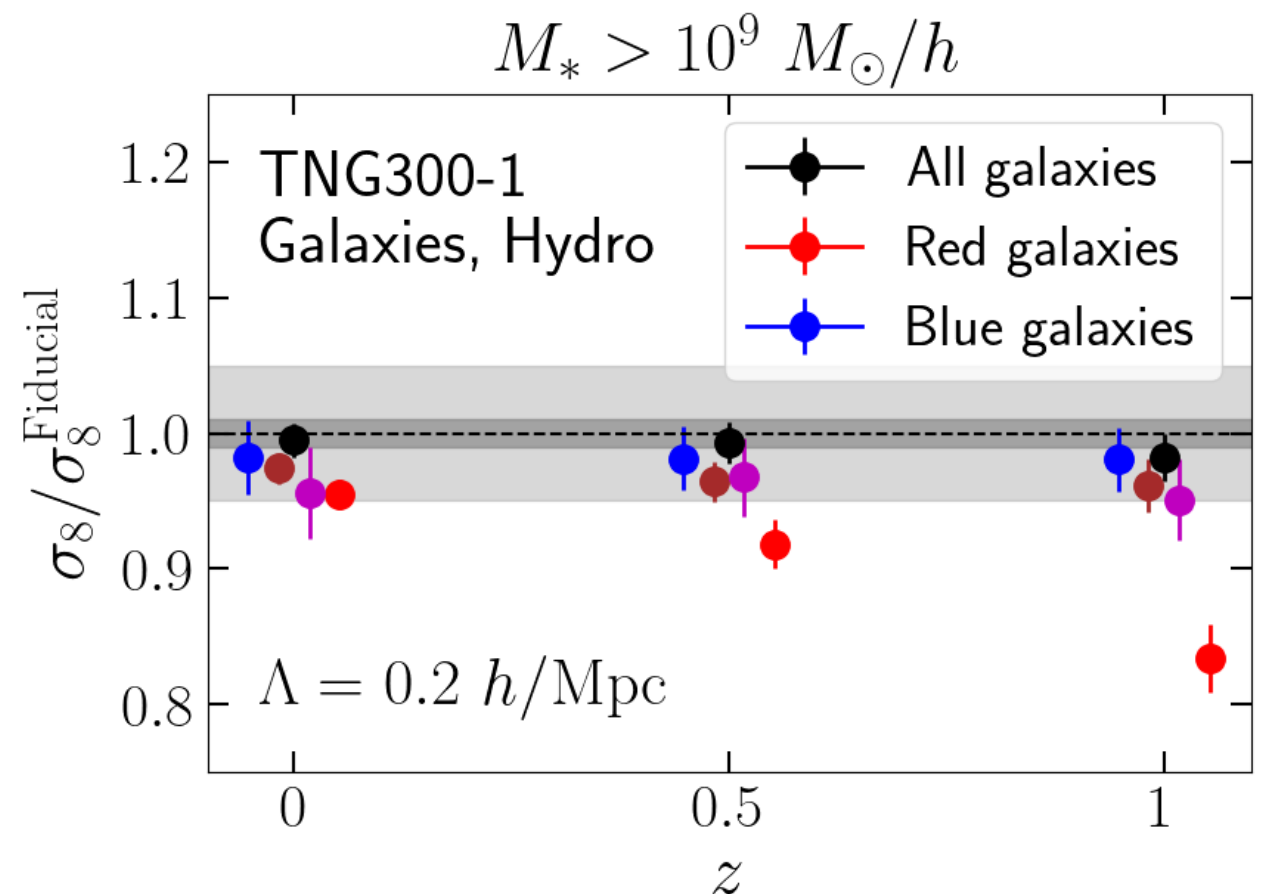


Also works for (simulated) galaxies

- Apply the same analysis to stellar-mass-selected galaxies in IllustrisTNG:

$L_{\text{box}} = 300 \text{ Mpc}/h$

No chance to do this using power spectrum+bispectrum due to cosmic variance...



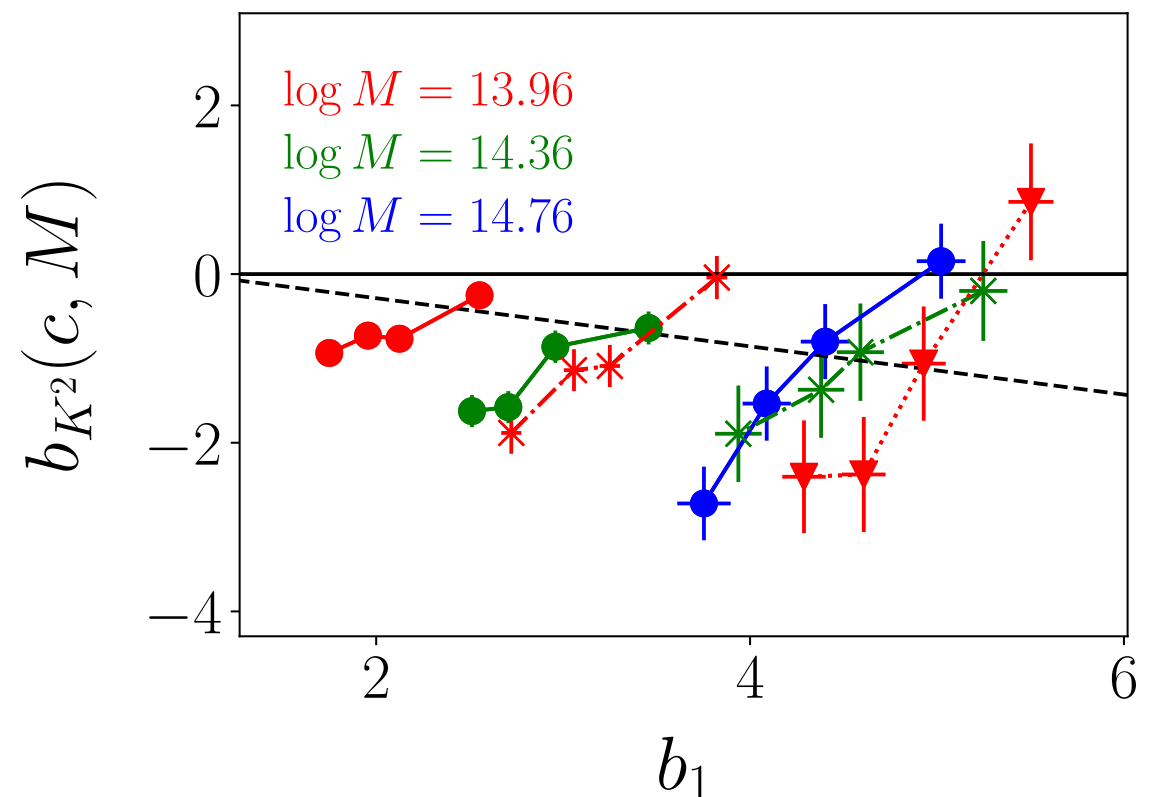
Ideal for studies of assembly bias

- Fixed-initial-condition analysis allows for precise measurements of bias parameters
- For example: detecting strong *assembly bias* in tidal bias coefficient

Colors: mass bins

Points: bias measured for 4 concentration quantiles

Dashed line: “coevolution” relation



BAO inference

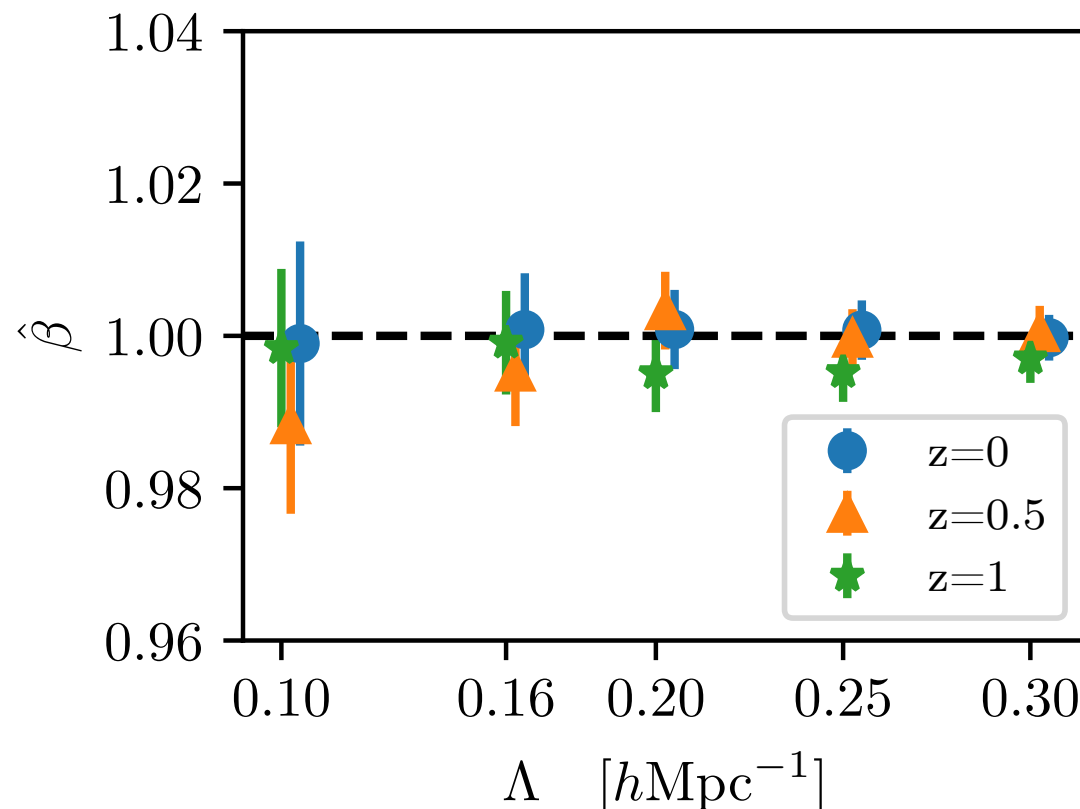
- Lagrangian perturbation theory based forward model incorporates precise local evolution of **BAO scale**
- Fixed phases (IC) -> *best possible BAO reconstruction*
- Trick: shift BAO scale in IC by rescaling:

$$f^2(k, \beta) = \frac{P_L(k, \beta)}{P_{\text{fid}}(k)} = \frac{1 + A \sin(k\beta r_{\text{fid}}) \exp(-k/k_D)}{1 + A \sin(k r_{\text{fid}}) \exp(-k/k_D)}$$

BAO inference

- Lagrangian perturbation theory based forward model incorporates precise local evolution of **BAO scale**
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β : ratio of inferred to fiducial isotropic BAO scale



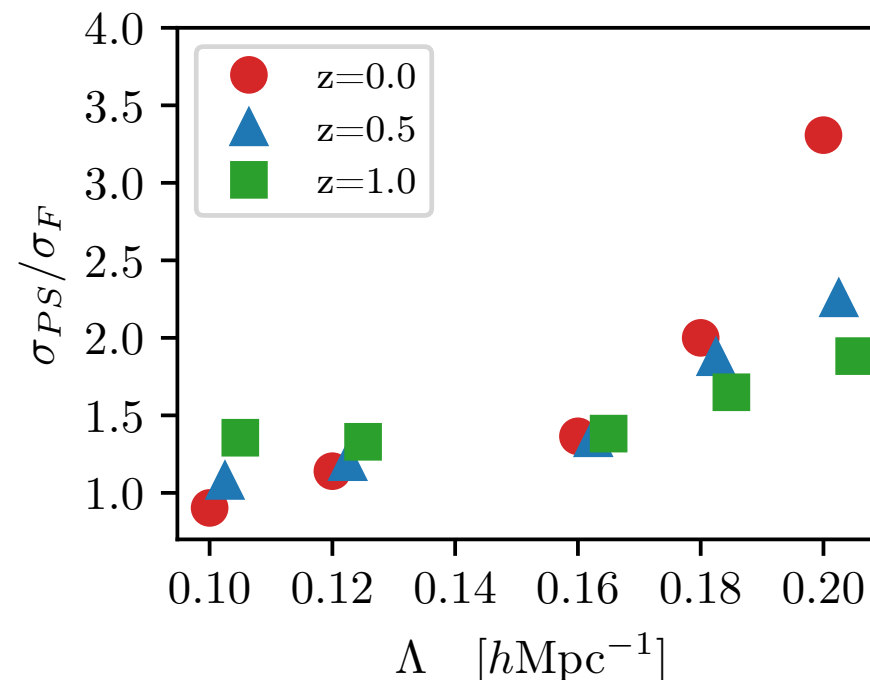
BAO inference

- Lagrangian perturbation theory based forward model incorporates precise local evolution of **BAO scale**
- Fixed phases (IC) -> *best possible BAO reconstruction*
- Comparison with (pre-reconstruction) $P(k)$

Ratio of **error bars on inferred BAO scale: $P(k)$** over field-level EFT

Both for fixed phases.

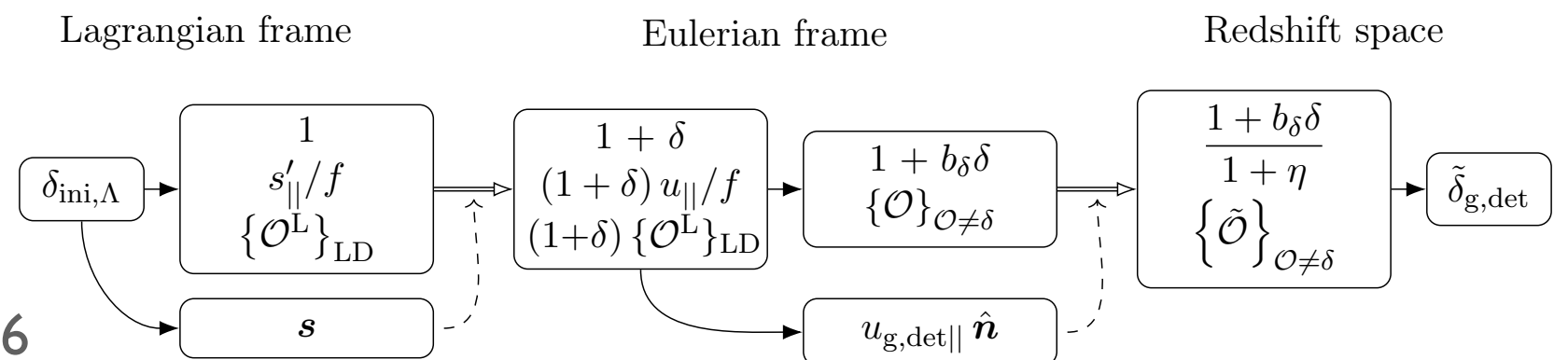
Factor 1.5-3 improvement over no-reconstruction $P(k)$ analysis.



(a) $\log_{10}(M/h^{-1}M_{\odot}) = 12.5 - 13.0$

Redshift-space distortions

- Notoriously difficult/cumbersome
- In forward model, can perform transformation to redshift space *non-perturbatively*



Redshift-space distortions

- Notoriously difficult/cumbersome
- In forward model, can perform transformation to redshift space *non-perturbatively*

- Velocity bias important:

$$u_{\text{g,det}||}(\mathbf{x}, \tau) = u_{||}(\mathbf{x}, \tau) + \sum_{\mathcal{U}} \beta_{\mathcal{U}}(\tau) \mathcal{U}(\mathbf{x}, \tau)$$

- In standard approach, absorbed in higher-derivative contributions, but not sufficiently accurate

Lagrangian frame

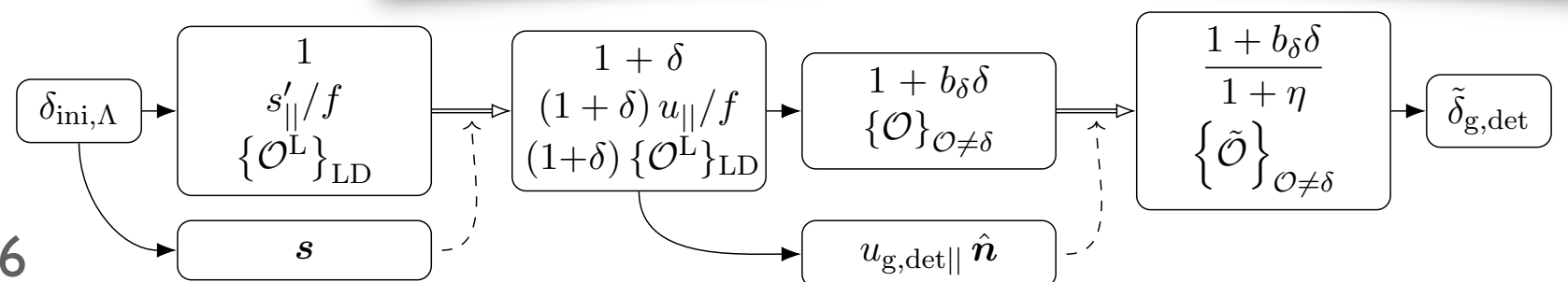
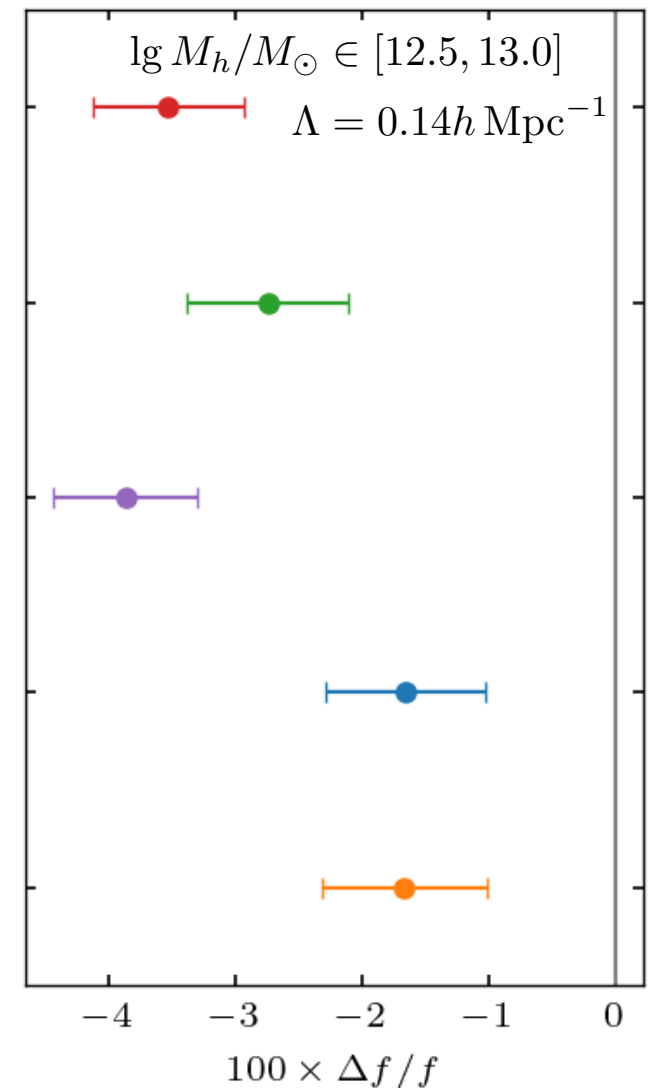
velocity-induced bias at leading-order

velocity-induced bias with higher-order contributions

velocity bias at leading-order

baseline

velocity bias at next-to-leading order



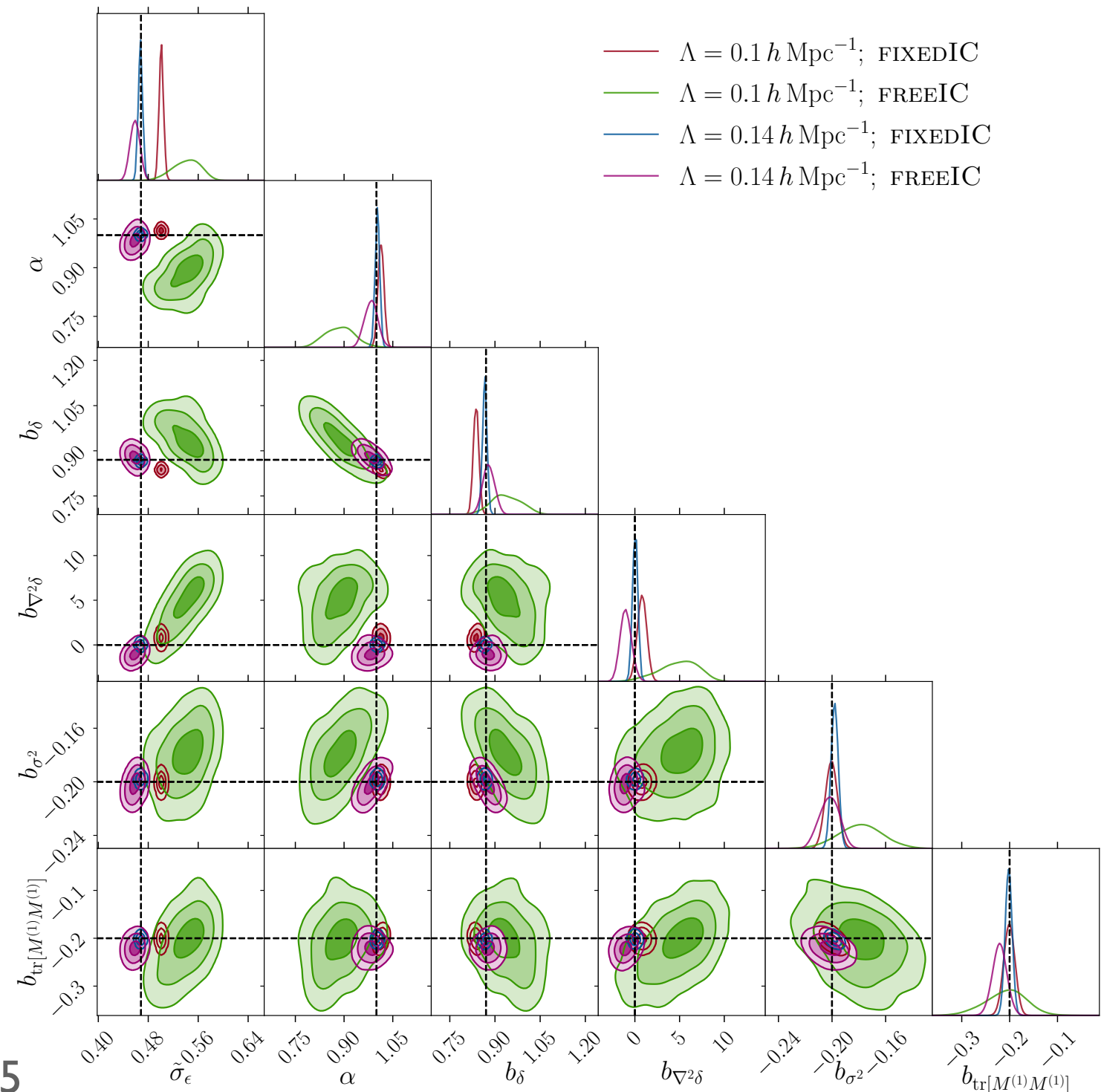
How much information is there actually?

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- **Full HMC sampling results** for mocks generated with the same model, but *different cutoffs* $\Lambda_{\text{mock}} > \Lambda_{\text{inference}}$

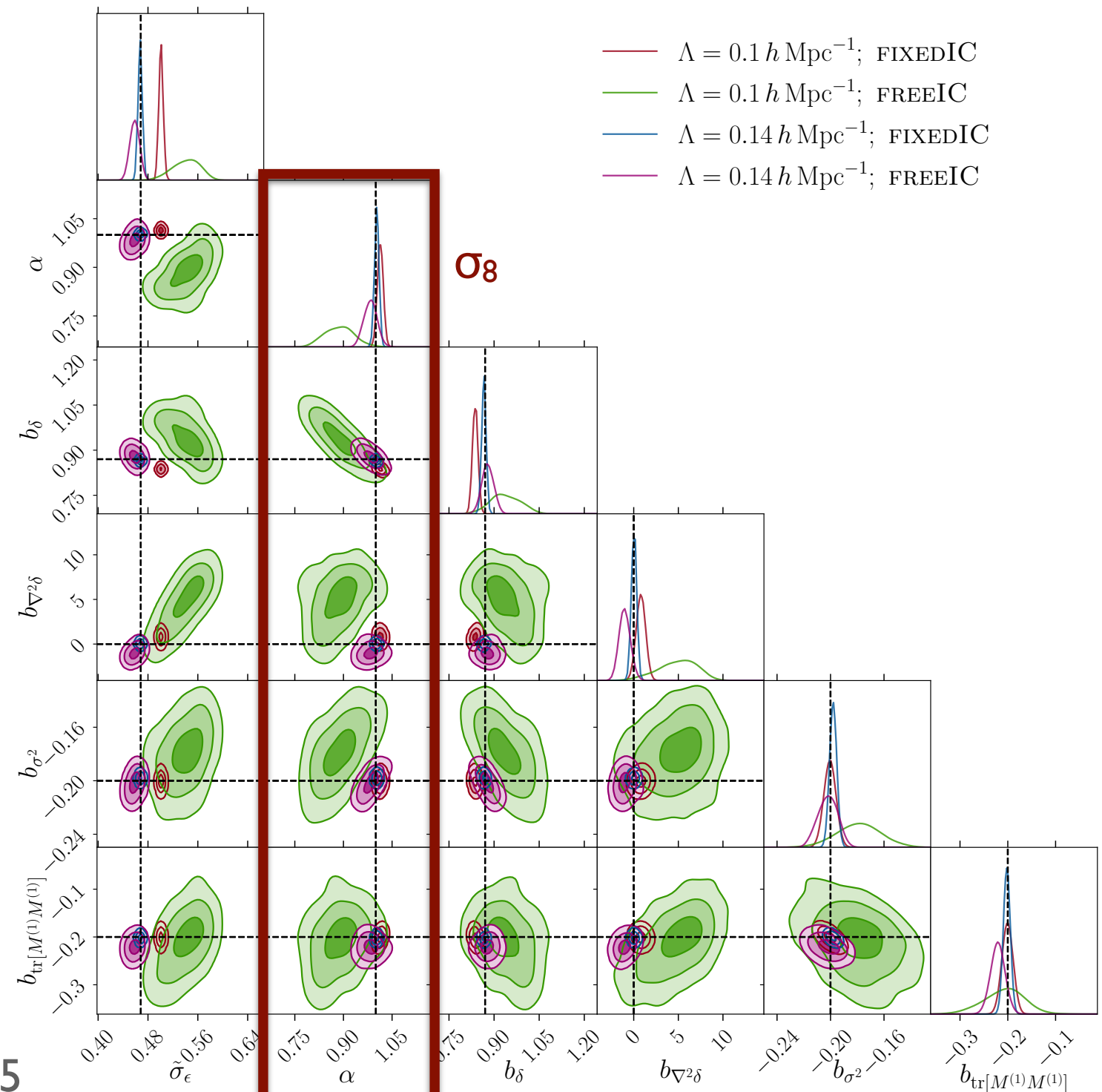
$$\frac{d}{d\Lambda} = \text{[diagram showing a sum of two terms: a single vertex and a pair of vertices connected by a line]}$$



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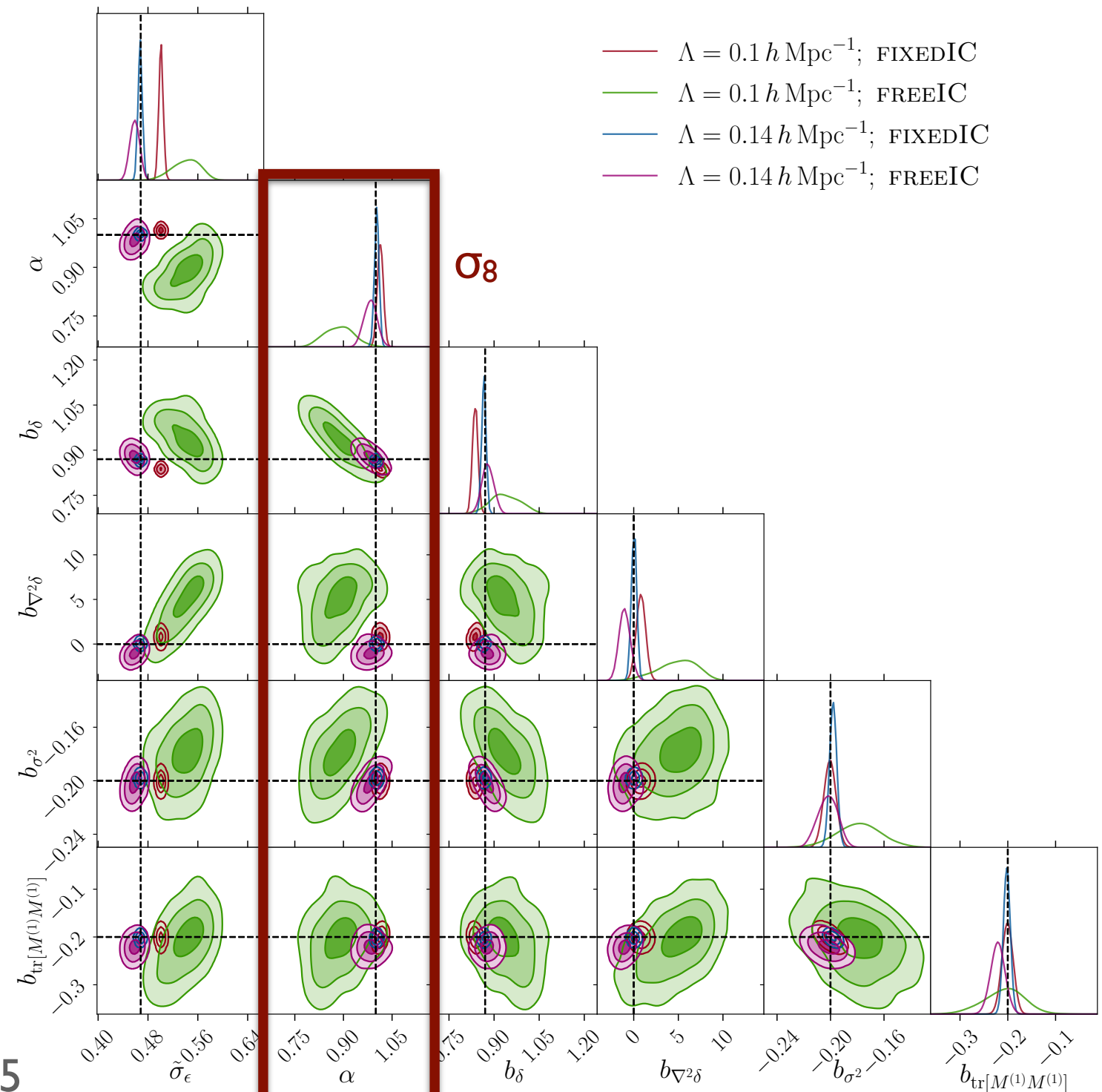
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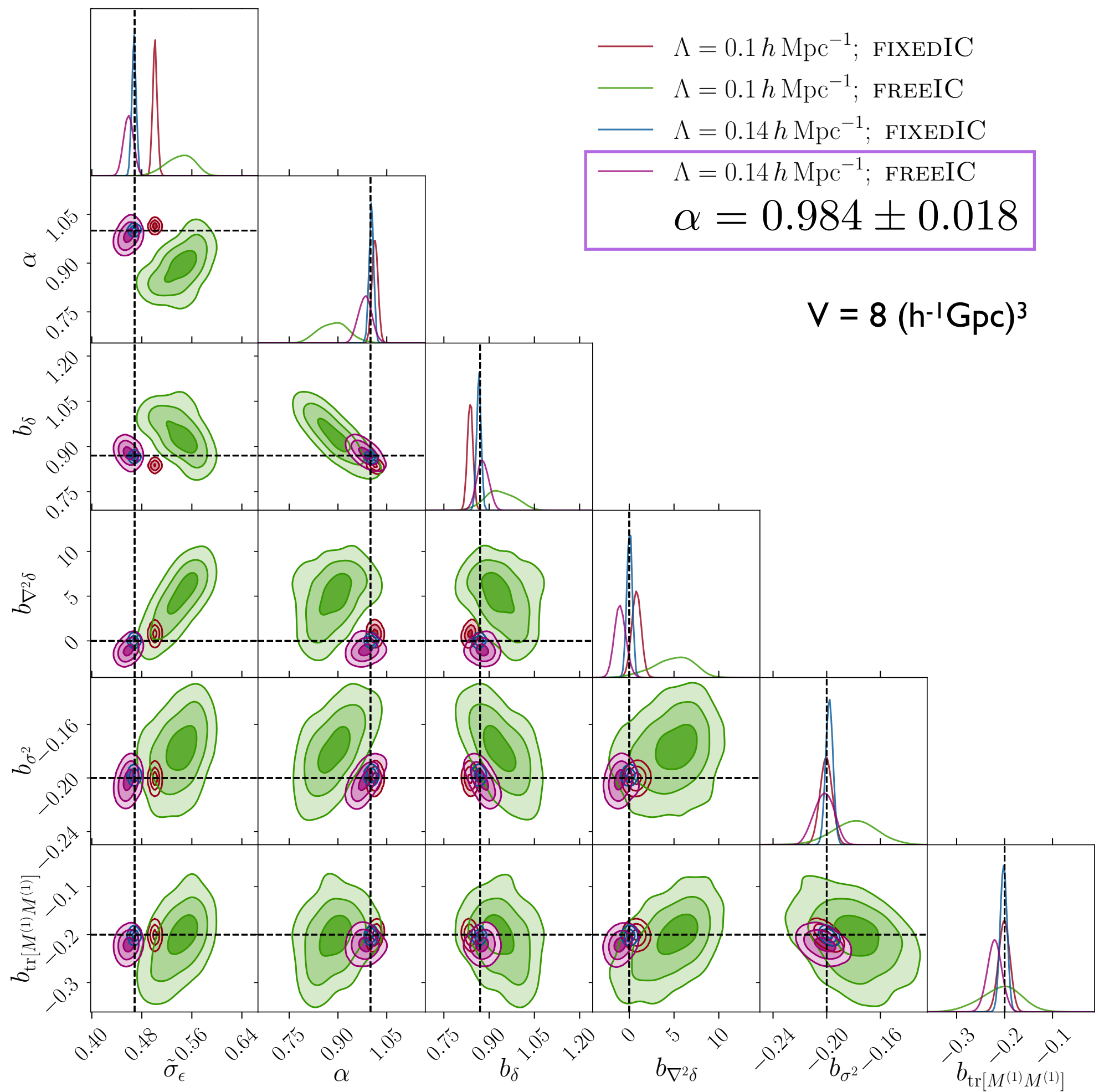
$$\frac{d}{d\Lambda} = \frac{d}{d\Lambda} + \frac{d}{d\Lambda} + \frac{d}{d\Lambda}$$



How much information is there actually?

- How much does the tiny, $\Delta\sigma_8 \ll 1\%$ error blow up once we marginalize over phases (initial conditions)?
- **Full HMC sampling results** for mocks generated with the same model, but *different cutoffs* $\Lambda_{\text{mock}} > \Lambda_{\text{inference}}$
- Fisher forecast indicates that σ_8 ($\Leftrightarrow \alpha$) constraint *improves by factor ~ 5 over power spectrum+bispectrum* ($\Lambda=0.14$)





Conclusions

- Two main messages:
 - *We can deal with complexities of galaxies rigorously on large scales -> **EFT***
 - *There is much more (trustable) information in galaxy clustering than what we are using so far -> **full inference* and/or new data summaries***

* For full disclosure: we still see issues when applying to fully nonlinear tracers (halos and high- Λ mocks).