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Kinetic Field Theory – frequently asked questions and answers

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with T. Daus, R. Lilow, N. Reinhardt, M. Bartelmann and many others

- Where is linear growth hidden in KFT?
- What is the difference to SPT?
- Is is possible to implement modified gravity theories in the KFT framework?

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Where is linear growth hidden in KFT?

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Does one get linear growth from purely Newtonian particle dynamics?

Recover linear growth (and beyond)

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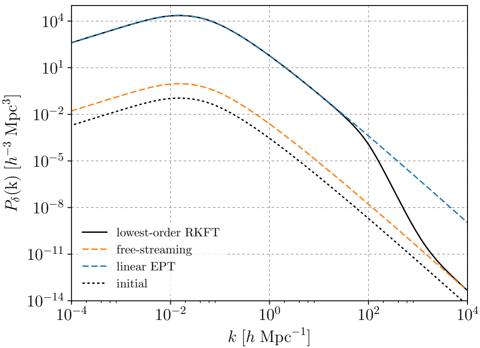


R. Lilow et al. 2019

N initially correlated particles

↓

Hamiltonian e.o.m with

$$H = H_0 + H_I$$

↙ ↘

$$q_0(t) = q^{(i)} + p^{(i)}(t - t^{(i)})$$

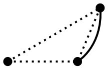
$$V(q) = \sum_{i,j} \frac{1}{|q_i - q_j|}$$

$$t = \int \frac{da}{a^3 H}$$

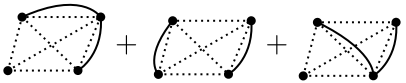
Recover linear growth

0 :

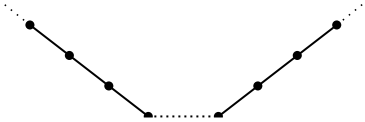
1 :



2 :



linear:



Microscopic particle system

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$$Z = \int d\Gamma_i \int \mathcal{D}\Phi e^{iS_0[\Phi]+iS_I}$$

$$iS_I = -\frac{i}{2}\Phi \cdot \sigma \cdot \Phi$$

Klimontovich phase-space density and response field:

$$\Phi = \begin{pmatrix} \Phi_\rho \\ \Phi_B \end{pmatrix} \propto \begin{pmatrix} \sum_j \delta_D(x - x_j) \\ \sum_j \chi_{p_j} \nabla_q \delta_D(x - x_j) \end{pmatrix}$$

,

$$\sigma \propto \begin{pmatrix} 0 & v(r) \\ v(r) & 0 \end{pmatrix}$$

Hubbard-Stratonovich transformation:

HS converts microscopic particle theory into macroscopic field theory

$$e^{-\frac{i}{2}\Phi\cdot\sigma\cdot\Phi} = \mathcal{N}\int\mathcal{D}\psi\, e^{i\psi\cdot\sigma^{-1}\cdot\psi - i\psi\cdot\Phi}$$

Macroscopic field theory

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introduce macroscopic fields:

$\psi = (\psi_{\mathrm{B}}, \psi_\rho)$, conjugate to $\Phi = (\Phi_\rho, \Phi_{\mathrm{B}})$

Macroscopic field theory

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$$\mathrm{Z} = \mathcal{N} \int \mathrm{d}\Gamma_\mathrm{i} \int \mathcal{D}\Phi \, \mathrm{e}^{\mathrm{i}S_0[\Phi]} \int \mathcal{D}\psi \, \mathrm{e}^{\mathrm{i}\psi\cdot\sigma^{-1}\cdot\psi - \mathrm{i}\psi\cdot\Phi}$$

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Macroscopic field theory

introduce macroscopic fields:

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$$= \tilde{\mathcal{N}} \int \mathrm{d}\Gamma_{\mathrm{i}} \int \mathcal{D}\Phi\, \mathrm{e}^{\mathrm{i}S_0[\Phi]} \int \mathcal{D}\psi_{\mathrm{B}}\, \mathcal{D}\tilde{\psi}_\rho\, \mathrm{e}^{\mathrm{i}\tilde{\psi}_\rho\cdot\psi_{\mathrm{B}} - \frac{\mathrm{i}}{2}\tilde{\psi}_\rho\cdot \mathrm{v}\cdot\Phi_{\mathrm{B}} - \mathrm{i}\psi_{\mathrm{B}}\cdot\Phi_\rho}$$

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$$Z = \tilde{\mathcal{N}} \underbrace{\int \mathcal{D}\psi \, e^{-i\psi_\rho \cdot \psi_B}}_{\text{macroscopic}} \underbrace{\int d\Gamma_i \int \mathcal{D}\Phi \, e^{iS_0[\Phi]} e^{\frac{i}{2}\psi_\rho \cdot v \cdot \Phi_B + i\psi_B \cdot \Phi_\rho}}_{\text{microscopic } Z_\psi}$$

$$= \tilde{\mathcal{N}} \int \mathcal{D}\psi \, e^{-i\psi_\rho \cdot \psi_B - iW_\psi}$$

$$W_\psi := \ln(Z_\psi)$$

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$$W_\psi \propto \sum_{n_\beta, n_\rho = 0}^{\infty} (\psi_B)^{n_\beta} \, (\psi_\rho)^{n_\rho} (\sigma_{\rho B})^{n_\beta} G^{(0)}_{\rho \dots \rho B \dots B}(1, \dots, n_\beta, 1', \dots, n'_\rho)$$

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$$W_\psi \propto 1 + \psi_B \, \sigma_{\rho B} G^{(0)}_{BB} \, \psi_B + \psi_\rho \, G^{(0)}_{\rho\rho} \, \psi_\rho + 2 \, \psi_B \, \sigma_{\rho B} G^{(0)}_{\rho B} \, \psi_\rho + \psi_B \, \sigma_{\rho B} G^{(0)}_{\rho\rho B} \, \psi_\rho \psi_\rho + \ldots$$

split action:

$$\mathrm{S} \stackrel{!}{=} \mathrm{S}_{\mathrm{tree}} + \mathrm{S}_{\mathrm{vertex}}$$

$$\mathrm{iS}_{\mathrm{tree}} = -\frac{1}{2}\psi \cdot \mathrm{D}^{-1} \cdot \psi$$

$$\mathrm{Z}[\mathrm{M}] = \mathrm{e}^{\mathrm{i}\hat{\mathrm{S}}_{\mathrm{vertex}}} \int \mathcal{D}\psi \, \mathrm{e}^{-\frac{1}{2}\psi \cdot \mathrm{D}^{-1} \cdot \psi + \mathrm{iM} \cdot \psi}$$

$$Z[M] = e^{i\hat{S}_{\mathrm{vertex}}} \int \mathcal{D}\psi \, e^{-\frac{1}{2}\psi \cdot D^{-1} \cdot \psi + iM \cdot \psi}$$

$$G_{\rho\rho} = D_{\rho\rho} + \text{terms involving vertices}$$

$$D_{\rho\rho} = D_R \cdot G^{(0)}_{\rho\rho} \cdot D_A$$

resummed propagators D_R, D_A contain infinite orders of interactions!

To recover linear growth...

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$$Z[M] = e^{i\hat{S}_{\text{vertex}}} \int \mathcal{D}\psi \, e^{-\frac{1}{2}\psi\cdot D^{-1}\cdot\psi + iM\cdot\psi}$$

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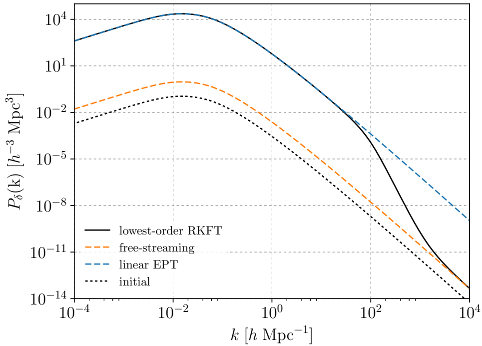
Expand initial conditions in terms of $P_\delta^{(i)}$

$$G^{(0)}_{\rho\rho} \propto P_\delta^{(i)} + P_\delta^{(i)} P_\delta^{(i)} + \ldots$$

$$\Rightarrow P_\delta(k,t) = \frac{D_+(t)}{D_+(t^i)} P_\delta^{(i)}(k)$$

Recover linear growth



But $\mathrm{G}_{\rho\rho}^{(0)}$ contains much more information, including mode-coupling effects!

What is the difference between KFT and SPT?

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Can KFT recover SPT results?

KFT recover linear growth exactly if...

we expand initial conditions in terms of $P_\delta^{(i)}$ to linear order!

$$Z[M] = \cancel{e^{i\hat{S}_{\text{vertex}}}} \int \mathcal{D}\psi \, e^{-\frac{1}{2}\psi\cdot D^{-1}\cdot\psi + iM\cdot\psi}$$

$$G_{\rho\rho} = D_{\rho\rho} \cancel{+\text{terms involving vertices}}$$

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Theory of cosmic structure formation

$$Z = \int \mathcal{D}\varphi \, P[\varphi]$$

Kinetic Field Theory:

- Initial conditions for N-particle ensemble
- Particles and their dynamics
- Hamiltonian e.o.m.

$$\vec{q}(t) = \vec{q}^{(i)} + g_{qp}(t, 0)\, \vec{p}^{(i)} + \int_0^t dt' \, g_{qp}(t, t') \, \vec{f}(t')$$

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Eulerian SPT:

- Averaged fields ρ and v
- Fluid equations

$$\partial_t \rho + \rho\,\theta + v^i\,\partial_i\,\rho = 0$$
$$\partial_t \theta + \partial_i \left(v^j\,\partial_j v^i\right) + 4\pi G\rho = 0$$

$(\theta = \partial_i v^i)$

single-stream approximation!

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Derive statistical properties by functional derivatives

Comparison of the perturbative expansion

Kinetic Field Theory:

- For

  $$\vec{q}(t) = \vec{q}^{(i)} + g_{qp}(t, 0)\, \vec{p}^{(i)}$$

  Z_{KFT} is exact.
- Free evolution contains non-linear effects due to initial correlations.
- Perturbative corrections are due to particle interactions.

Comparison of the perturbative expansion

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Eulerian SPT:

- Linear theory describes the independent evolution of Fourier field modes (includes gravitational interactions).
- By construction (SSA) linear theory is not exact.
- Perturbative corrections arise due coupling of the field modes by non-linear advective terms.

Comparison of the perturbative expansion

$$\mathrm{Z}[\mathrm{M}] = \mathrm{e}^{\mathrm{i}\hat{\mathrm{S}}_{\mathrm{vertex}}} \int \mathcal{D}\psi \, \mathrm{e}^{-\frac{1}{2}\psi\cdot \mathrm{D}^{-1}\cdot\psi + \mathrm{i}\mathrm{M}\cdot\psi}$$

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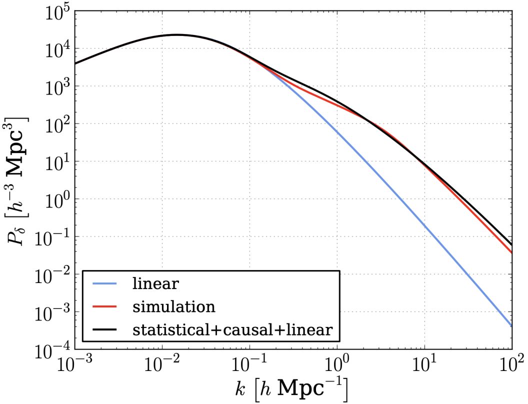
$$\mathrm{D}_{\rho\rho} = \mathrm{D}_{\mathrm{R}} \cdot \mathrm{G}^{(0)}_{\rho\rho} \cdot \mathrm{D}_{\mathrm{A}}$$

resummed propagators D_{R}, D_{A} contain infinite orders of interactions!

Recover 1loop SPT if...

we expand initial conditions in terms of $P_\delta^{(i)}$ up to second order.



from R. Lilow, PhD thesis

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- To recover SPT results we have to neglect higher-order correlations.
- Does this pattern continue?

Is is possible to implement modified gravity theories in the KFT framework?

How to modify gravity

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modified Poisson equation

$$\nabla^2\Psi = \frac{a^2}{2}\left(1+\frac{\Delta G_{eff}}{G}\right)\kappa^2\delta\rho_m$$

use parametrisation

$$\frac{G_{eff}}{G} = A + \sum_i^{N_0} B_i \prod_j^{N_i} b_{ij}\left(\frac{r}{r_{0ij}}\right)^{a_{ij}}\left\{\left[1+\left(\frac{r_{0ij}}{r}\right)^{a_{ij}}\right]^{1/b_{ij}} - 1\right\}$$

Lombriser et al. 2016

nDGP in the mean-field approximation of KFT

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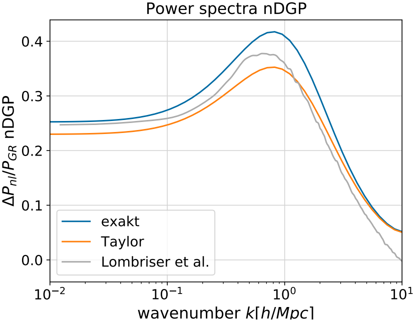
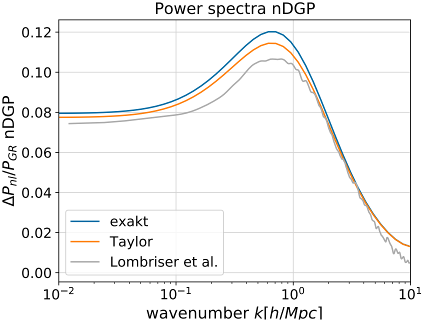
$r_c = 0.5,\ k_* = 1.0$



$r_c = 2.0,\ k_* = 0.8$



from N.Reinhardt, Master thesis 2023