

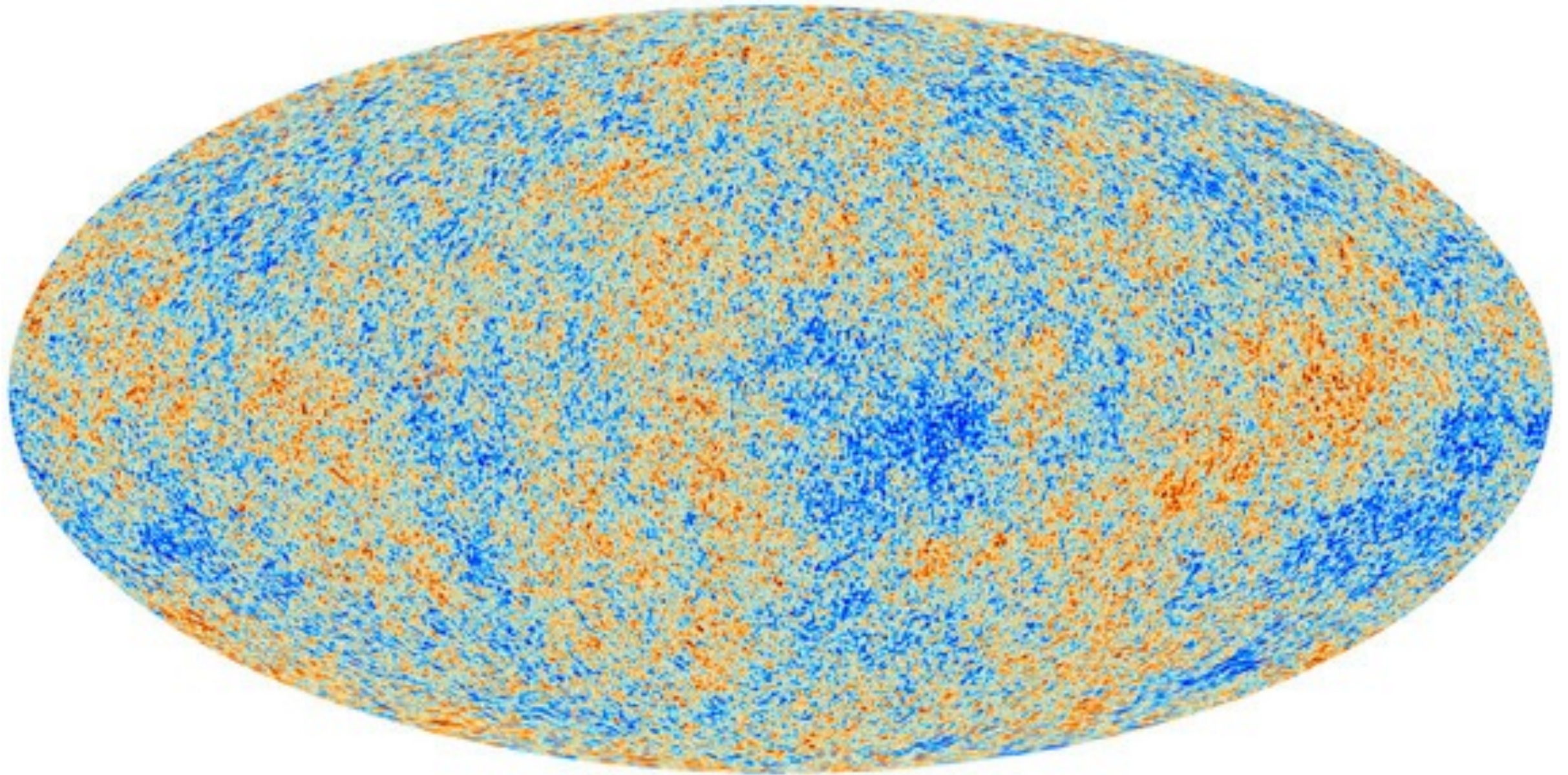
# Cosmological Data Analysis for the 21st Century

Pedro G. Ferreira



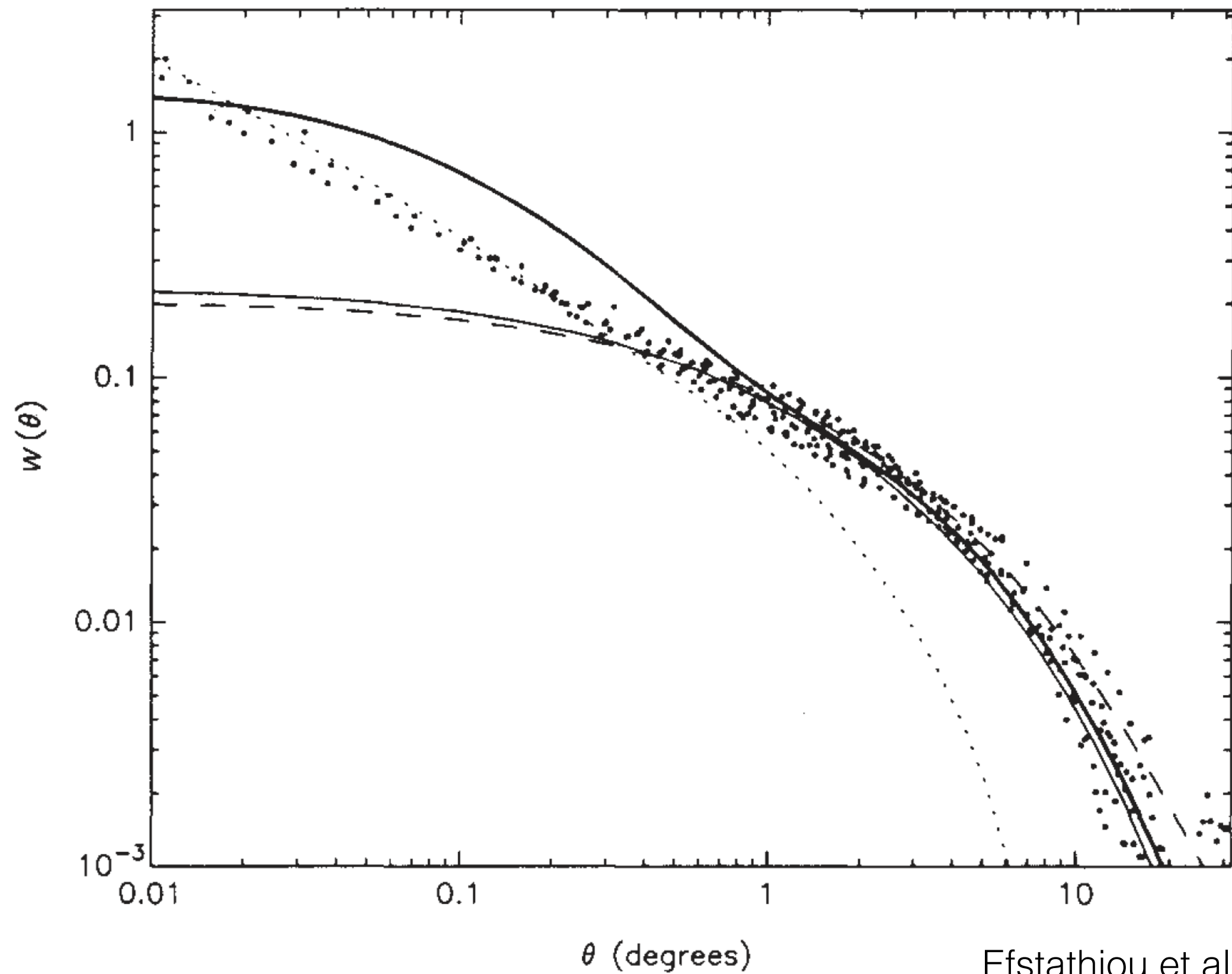
Prague, 2023

CMB today

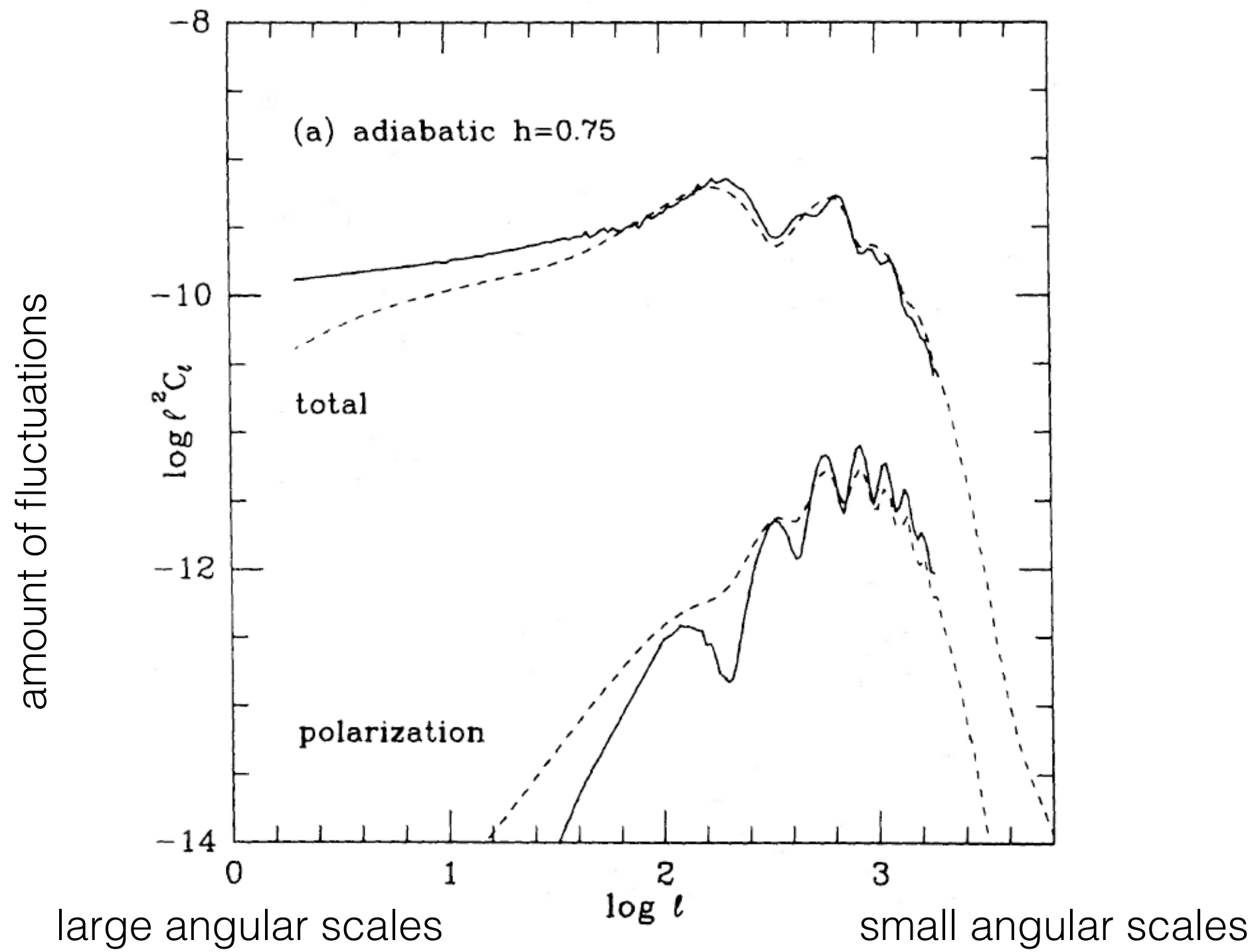


Planck 2015

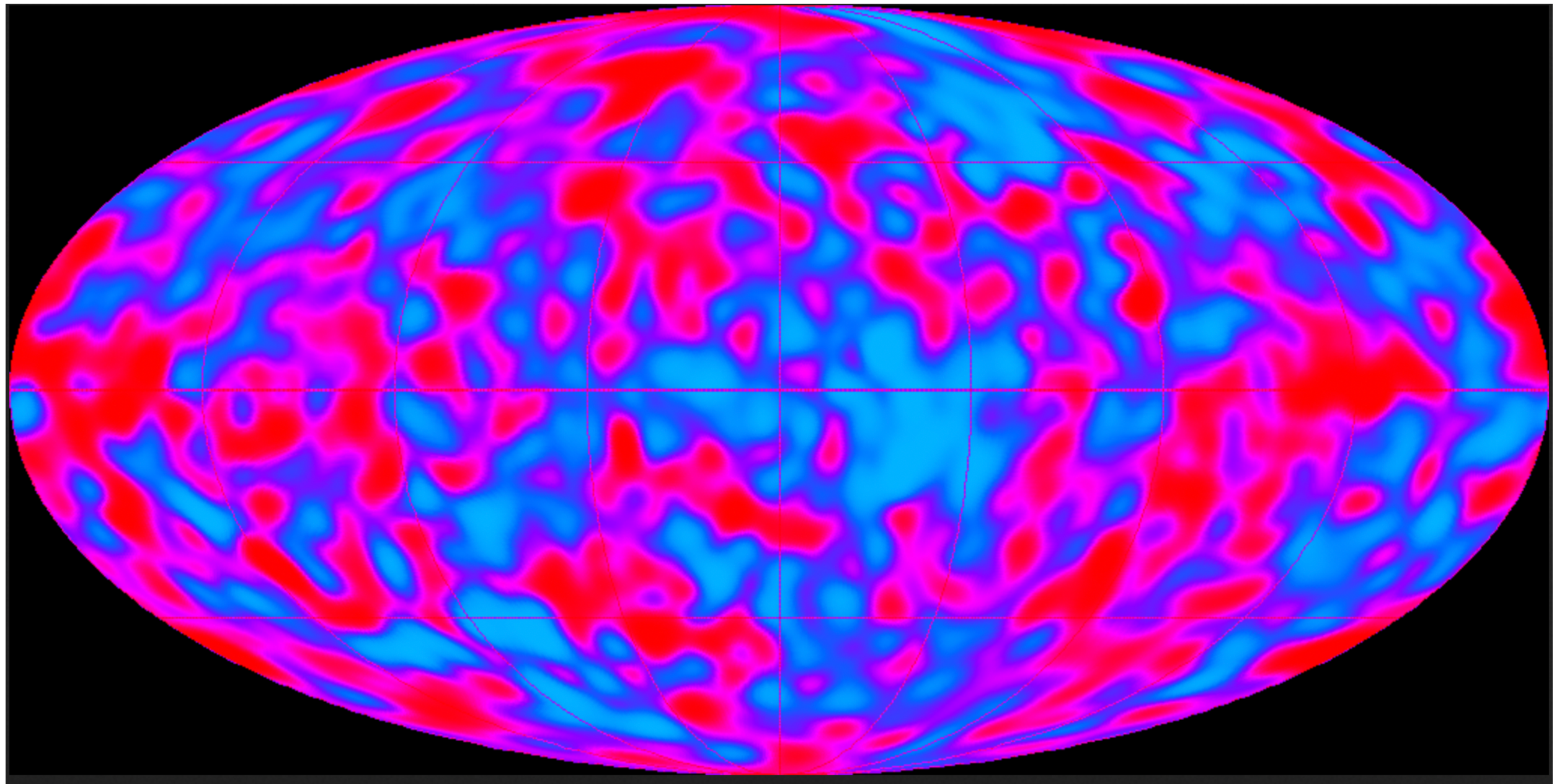
The 1990s



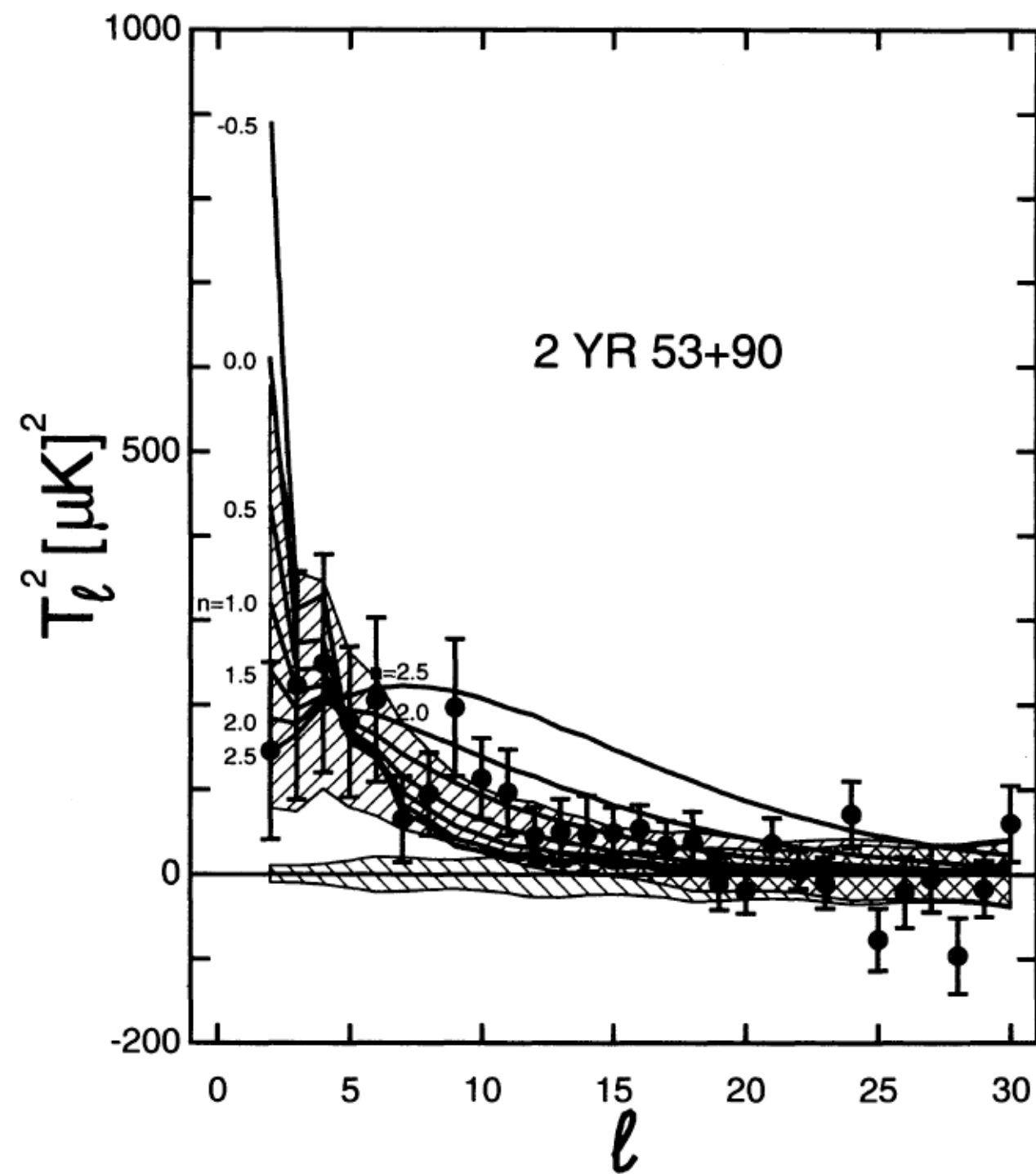
Efstathiou et al 199



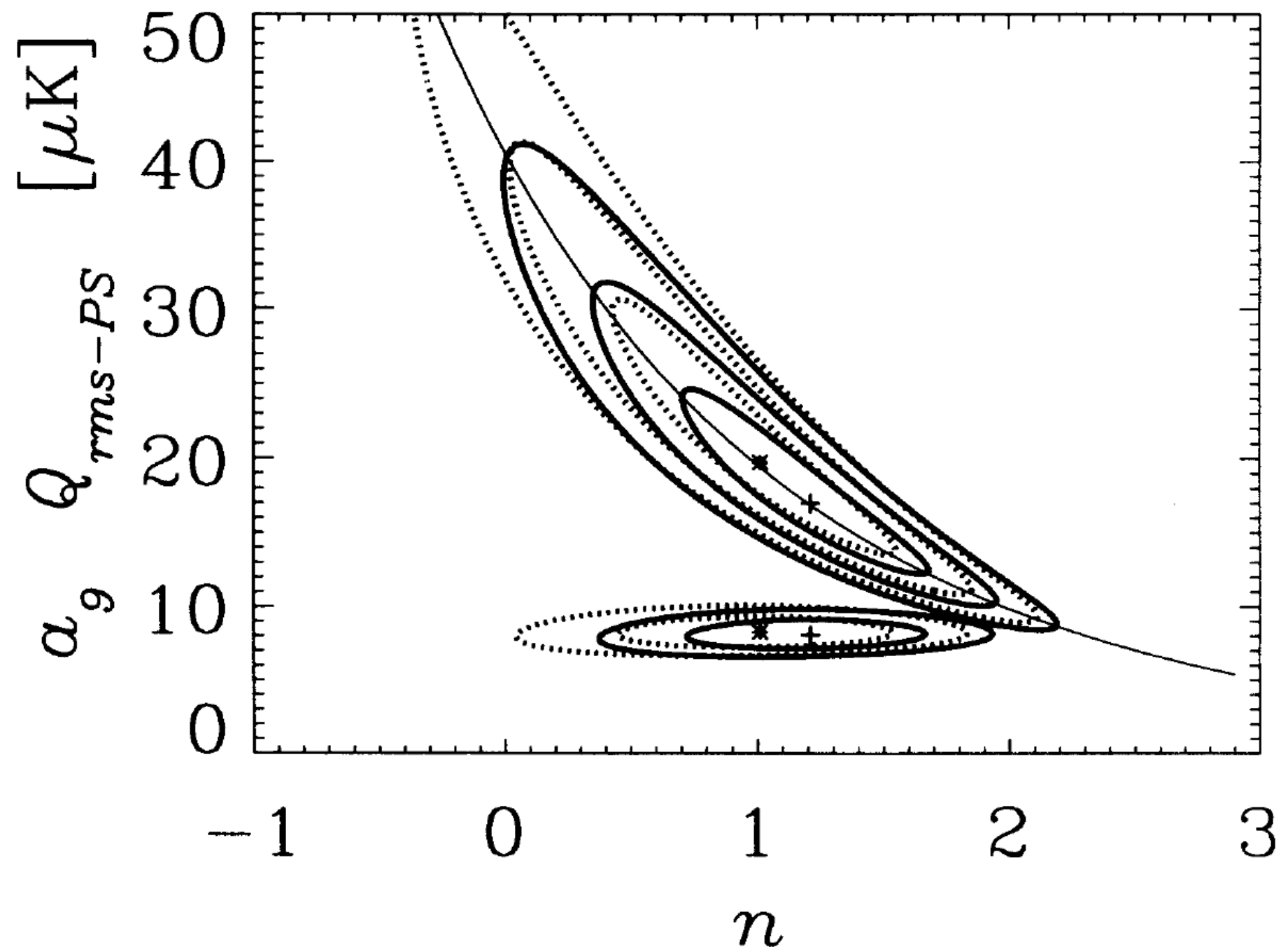
Bond & Efstathiou 1987



COBE 1992



Wright *et al* (COBE) 1994



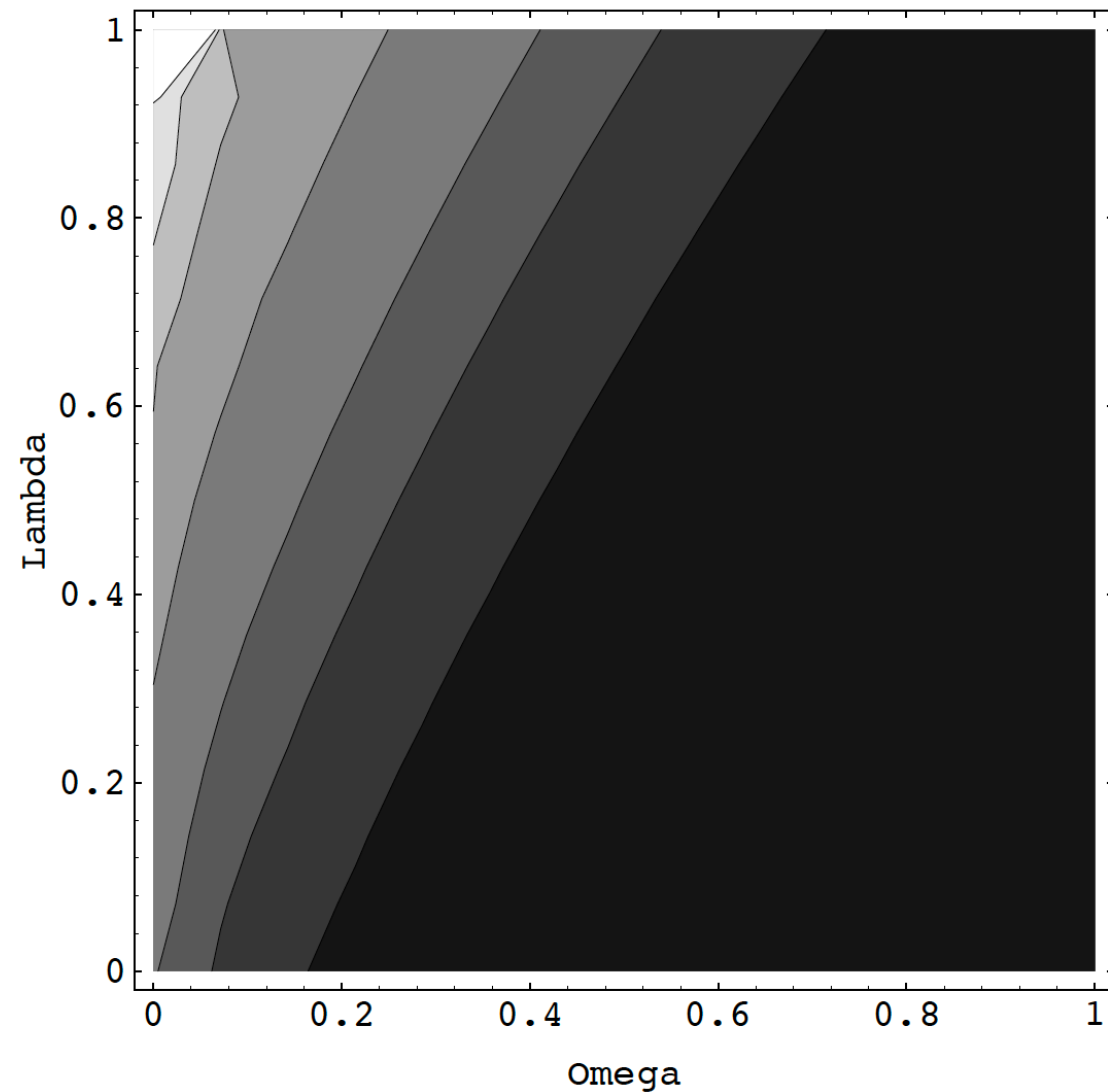
Gorski *et al* (COBE) 1994

# Models and Evidence circa 1995

## $H_0$ and Odds on Cosmology

Andrew Jaffe

Canadian Institute for Theoretical Astrophysics,  
60 St. George St., Toronto, Ontario M5S 1A1, Canada



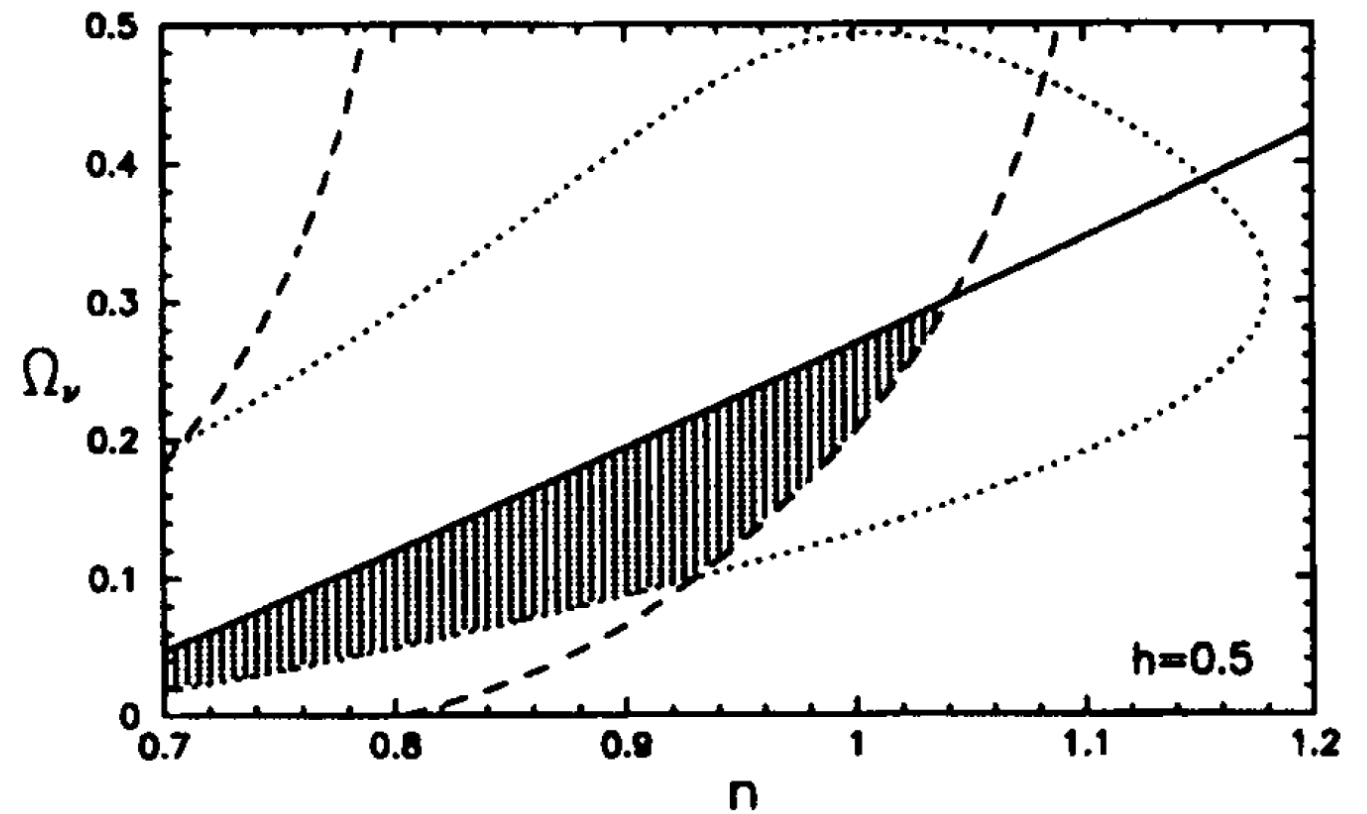
## Pursuing parameters for critical-density dark matter models

Andrew R. Liddle,<sup>1</sup> David H. Lyth,<sup>2</sup> R. K. Schaefer,<sup>3</sup> Q. Shafi<sup>3</sup>  
and Pedro T. P. Viana<sup>1</sup>

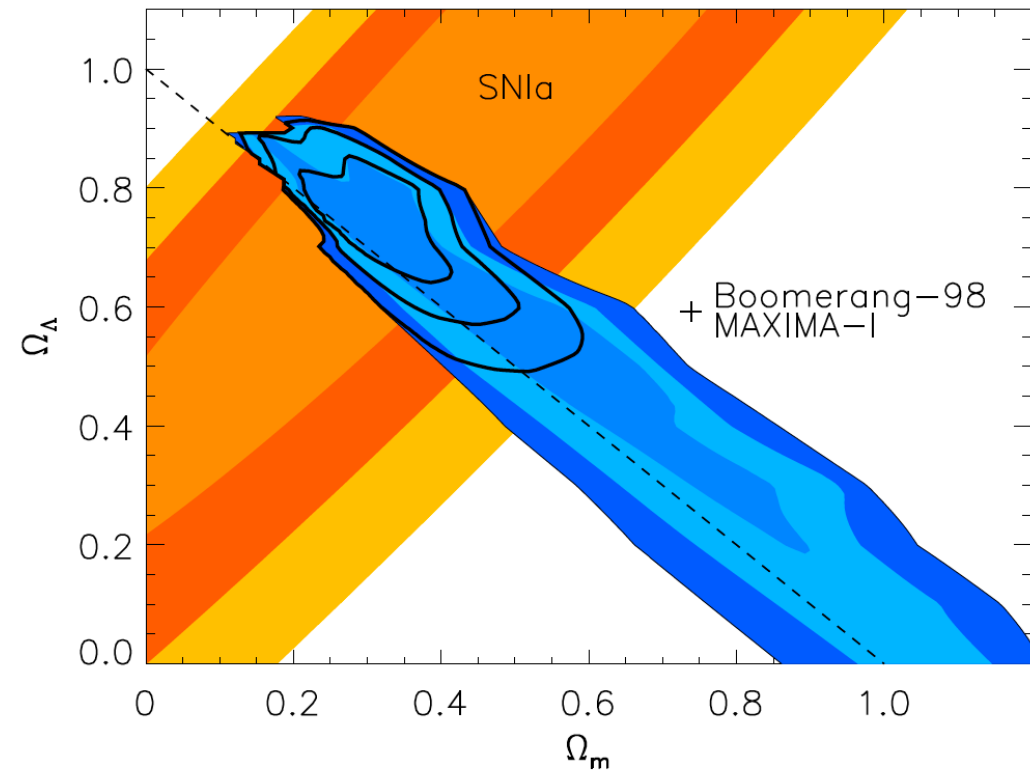
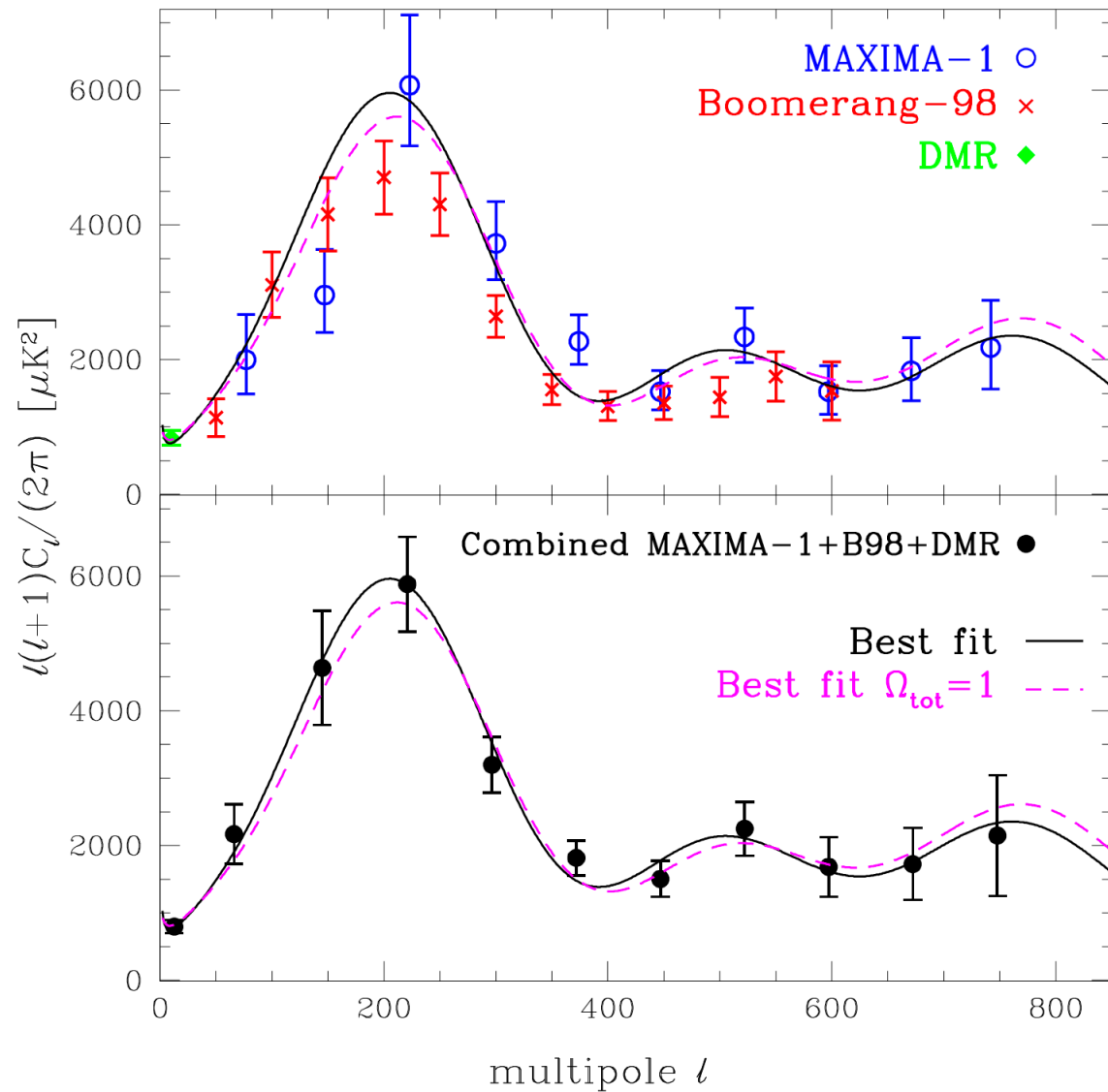
<sup>1</sup>*Astronomy Centre, University of Sussex, Falmer, Brighton BN1 9QH*

<sup>2</sup>*School of Physics and Chemistry, University of Lancaster, Lancaster LA1 4YB*

<sup>3</sup>*Bartol Research Institute, University of Delaware, Newark, Delaware 19716, USA*



# Models and Evidence circa 2000



|                                | $\Omega_{\text{tot}}$  | $\Omega_b h^2$            | $n_s$                  | $\Omega_c h^2$         |
|--------------------------------|------------------------|---------------------------|------------------------|------------------------|
| B98+DMR                        | $1.15^{+0.10}_{-0.09}$ | $0.036^{+0.006}_{-0.005}$ | $1.04^{+0.10}_{-0.09}$ | $0.24^{+0.08}_{-0.09}$ |
| MAXIMA-1+DMR                   | $1.01^{+0.09}_{-0.09}$ | $0.031^{+0.007}_{-0.006}$ | $1.06^{+0.10}_{-0.09}$ | $0.18^{+0.07}_{-0.06}$ |
| B98+MAXIMA-1+DMR               | $1.11^{+0.07}_{-0.07}$ | $0.032^{+0.005}_{-0.005}$ | $1.01^{+0.09}_{-0.08}$ | $0.14^{+0.06}_{-0.05}$ |
| +( $\Omega_{\text{tot}} = 1$ ) | 1                      | $0.030^{+0.004}_{-0.004}$ | $0.99^{+0.07}_{-0.06}$ | $0.19^{+0.07}_{-0.06}$ |
| CMB+LSS                        | $1.11^{+0.05}_{-0.05}$ | $0.032^{+0.004}_{-0.004}$ | $1.00^{+0.09}_{-0.06}$ | $0.13^{+0.02}_{-0.01}$ |
| CMB+SNIa                       | $1.09^{+0.06}_{-0.05}$ | $0.032^{+0.005}_{-0.005}$ | $1.00^{+0.09}_{-0.08}$ | $0.10^{+0.04}_{-0.04}$ |
| CMB+LSS+SNIa                   | $1.06^{+0.04}_{-0.04}$ | $0.033^{+0.005}_{-0.004}$ | $1.03^{+0.09}_{-0.07}$ | $0.14^{+0.03}_{-0.02}$ |

MAXIMA & BOOMERANG 2001

# MCMC arrives in 2001!

## Bayesian Methods for Cosmological Parameter Estimation from Cosmic Microwave Background Measurements

Nelson Christensen<sup>1</sup>, Renate Meyer<sup>2</sup>, Lloyd Knox<sup>3</sup>, Ben Luey<sup>1</sup>



<sup>1</sup>Physics and Astronomy, Carleton College, Northfield, MN 55057, USA

<sup>2</sup>Department of Statistics, The University of Auckland, Auckland, New Zealand

<sup>3</sup>Department of Physics, University of California - Davis, Davis, CA 95616, USA

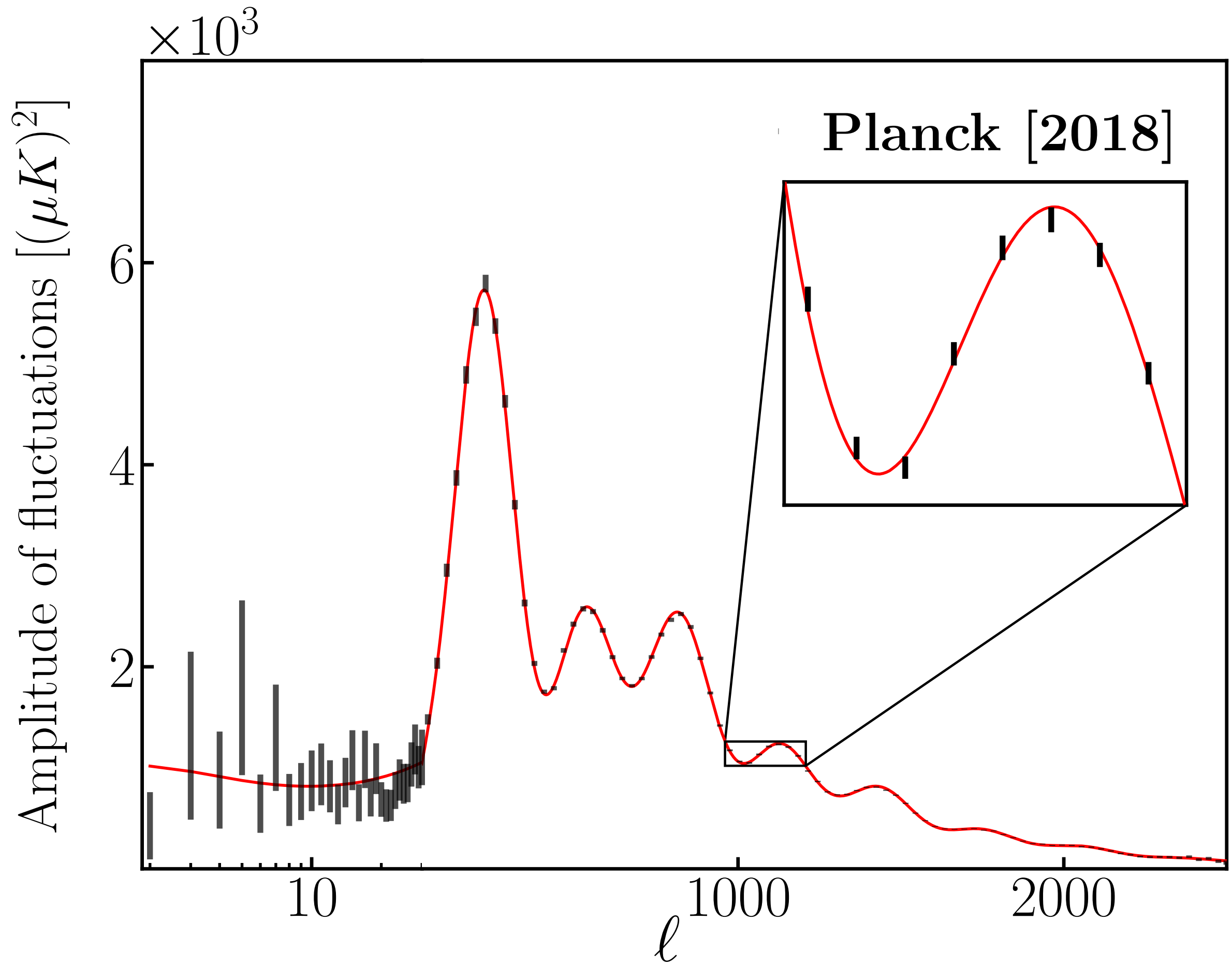
**Abstract.** We present a strategy for a statistically rigorous Bayesian approach to the problem of determining cosmological parameters from the results of observations of anisotropies in the cosmic microwave background. Our strategy relies on Markov chain Monte Carlo methods, specifically the Metropolis-Hastings algorithm, to perform the necessary high-dimensional integrals. We describe the Metropolis-Hastings algorithm in detail and discuss the results of our test on simulated data.

# The Methodological Revolution

1990s: The rise of Bayesian statistics

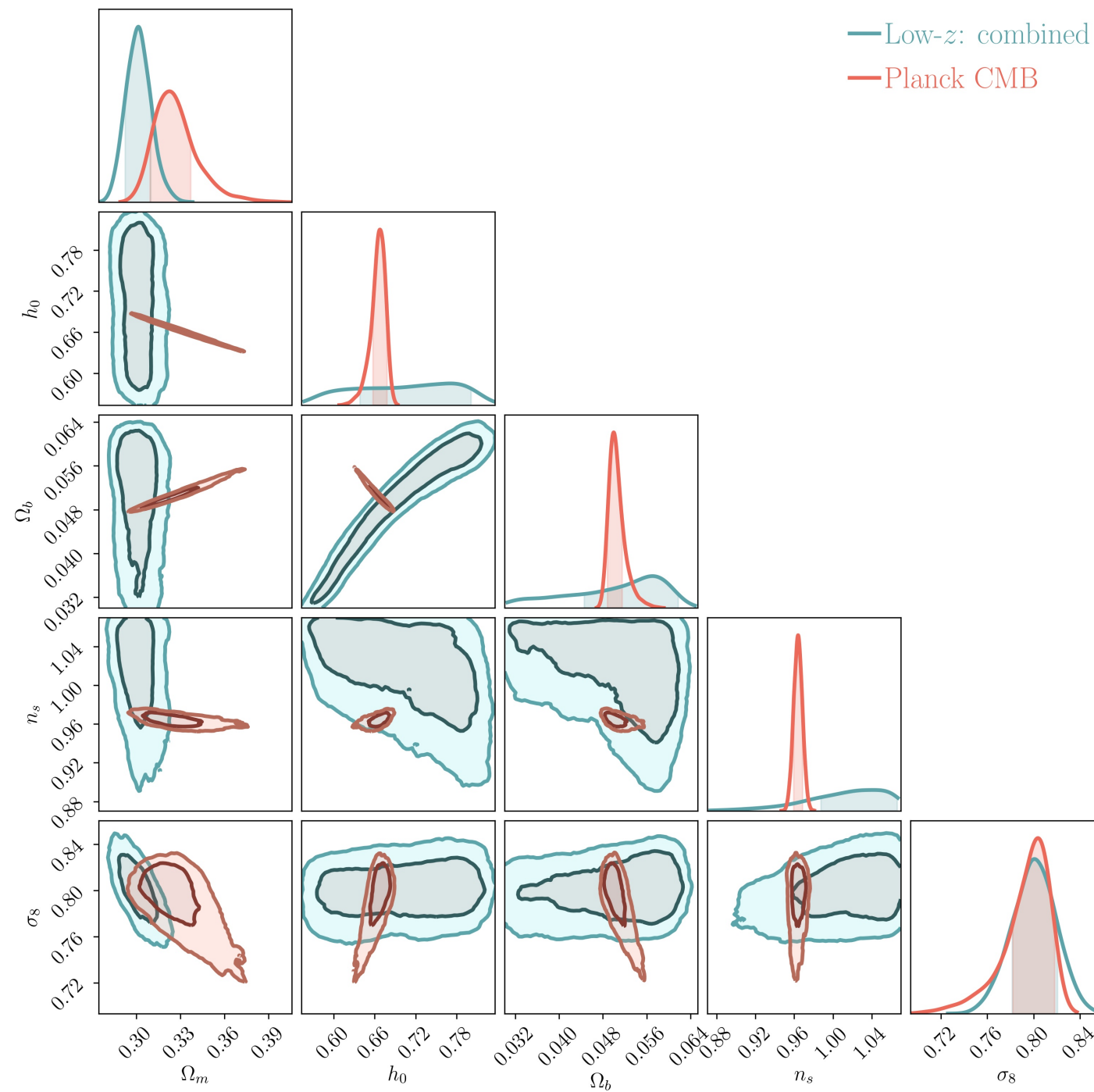
- Likelihoods
- Evidence
- MCMC
- Hierarchical Models
- etc ...

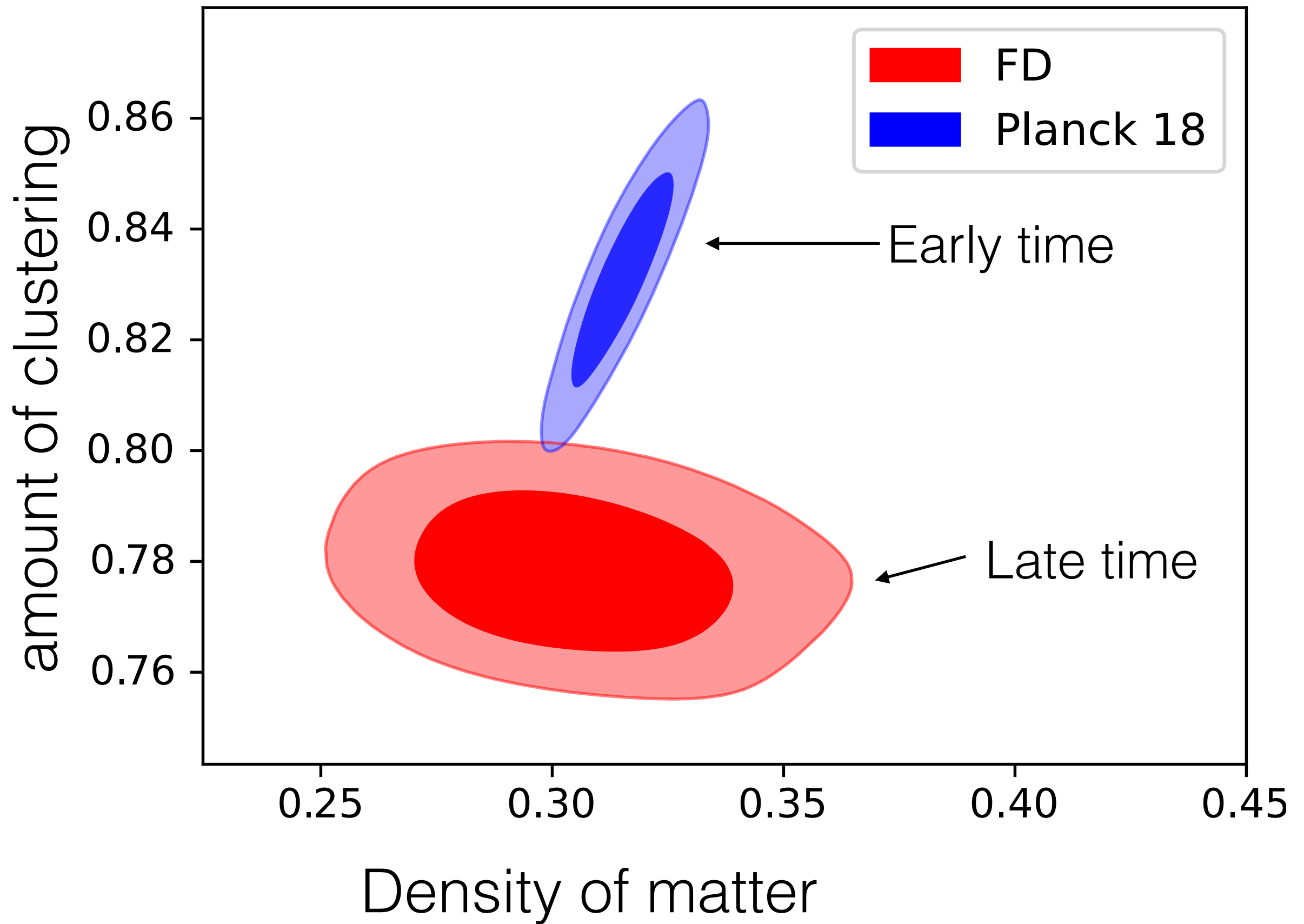
Now



| Parameter                                         | TT+lowE<br>68% limits | TE+lowE<br>68% limits     | EE+lowE<br>68% limits           | TT,TE,EE+lowE<br>68% limits  | TT,TE,EE+lowE+lensing<br>68% limits | TT,TE,EE+lowE+lensing+BAO<br>68% limits |
|---------------------------------------------------|-----------------------|---------------------------|---------------------------------|------------------------------|-------------------------------------|-----------------------------------------|
| $\Omega_b h^2$ . . . . .                          | $0.02212 \pm 0.00022$ | $0.02249 \pm 0.00025$     | $0.0240 \pm 0.0012$             | $0.02236 \pm 0.00015$        | $0.02237 \pm 0.00015$               | $0.02242 \pm 0.00014$                   |
| $\Omega_c h^2$ . . . . .                          | $0.1206 \pm 0.0021$   | $0.1177 \pm 0.0020$       | $0.1158 \pm 0.0046$             | $0.1202 \pm 0.0014$          | $0.1200 \pm 0.0012$                 | $0.11933 \pm 0.00091$                   |
| $100\theta_{\text{MC}}$ . . . . .                 | $1.04077 \pm 0.00047$ | $1.04139 \pm 0.00049$     | $1.03999 \pm 0.00089$           | $1.04090 \pm 0.00031$        | $1.04092 \pm 0.00031$               | $1.04101 \pm 0.00029$                   |
| $\tau$ . . . . .                                  | $0.0522 \pm 0.0080$   | $0.0496 \pm 0.0085$       | $0.0527 \pm 0.0090$             | $0.0544^{+0.0070}_{-0.0081}$ | $0.0544 \pm 0.0073$                 | $0.0561 \pm 0.0071$                     |
| $\ln(10^{10} A_s)$ . . . . .                      | $3.040 \pm 0.016$     | $3.018^{+0.020}_{-0.018}$ | $3.052 \pm 0.022$               | $3.045 \pm 0.016$            | $3.044 \pm 0.014$                   | $3.047 \pm 0.014$                       |
| $n_s$ . . . . .                                   | $0.9626 \pm 0.0057$   | $0.967 \pm 0.011$         | $0.980 \pm 0.015$               | $0.9649 \pm 0.0044$          | $0.9649 \pm 0.0042$                 | $0.9665 \pm 0.0038$                     |
| $H_0$ [km s <sup>-1</sup> Mpc <sup>-1</sup> ] . . | $66.88 \pm 0.92$      | $68.44 \pm 0.91$          | $69.9 \pm 2.7$                  | $67.27 \pm 0.60$             | $67.36 \pm 0.54$                    | $67.66 \pm 0.42$                        |
| $\Omega_\Lambda$ . . . . .                        | $0.679 \pm 0.013$     | $0.699 \pm 0.012$         | $0.711^{+0.033}_{-0.026}$       | $0.6834 \pm 0.0084$          | $0.6847 \pm 0.0073$                 | $0.6889 \pm 0.0056$                     |
| $\Omega_m$ . . . . .                              | $0.321 \pm 0.013$     | $0.301 \pm 0.012$         | $0.289^{+0.026}_{-0.033}$       | $0.3166 \pm 0.0084$          | $0.3153 \pm 0.0073$                 | $0.3111 \pm 0.0056$                     |
| $\Omega_m h^2$ . . . . .                          | $0.1434 \pm 0.0020$   | $0.1408 \pm 0.0019$       | $0.1404^{+0.0034}_{-0.0039}$    | $0.1432 \pm 0.0013$          | $0.1430 \pm 0.0011$                 | $0.14240 \pm 0.00087$                   |
| $\Omega_m h^3$ . . . . .                          | $0.09589 \pm 0.00046$ | $0.09635 \pm 0.00051$     | $0.0981^{+0.0016}_{-0.0018}$    | $0.09633 \pm 0.00029$        | $0.09633 \pm 0.00030$               | $0.09635 \pm 0.00030$                   |
| $\sigma_8$ . . . . .                              | $0.8118 \pm 0.0089$   | $0.793 \pm 0.011$         | $0.796 \pm 0.018$               | $0.8120 \pm 0.0073$          | $0.8111 \pm 0.0060$                 | $0.8102 \pm 0.0060$                     |
| $S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$ .       | $0.840 \pm 0.024$     | $0.794 \pm 0.024$         | $0.781^{+0.052}_{-0.060}$       | $0.834 \pm 0.016$            | $0.832 \pm 0.013$                   | $0.825 \pm 0.011$                       |
| $\sigma_8 \Omega_m^{0.25}$ . . . . .              | $0.611 \pm 0.012$     | $0.587 \pm 0.012$         | $0.583 \pm 0.027$               | $0.6090 \pm 0.0081$          | $0.6078 \pm 0.0064$                 | $0.6051 \pm 0.0058$                     |
| $z_{\text{re}}$ . . . . .                         | $7.50 \pm 0.82$       | $7.11^{+0.91}_{-0.75}$    | $7.10^{+0.87}_{-0.73}$          | $7.68 \pm 0.79$              | $7.67 \pm 0.73$                     | $7.82 \pm 0.71$                         |
| $10^9 A_s$ . . . . .                              | $2.092 \pm 0.034$     | $2.045 \pm 0.041$         | $2.116 \pm 0.047$               | $2.101^{+0.031}_{-0.034}$    | $2.100 \pm 0.030$                   | $2.105 \pm 0.030$                       |
| $10^9 A_s e^{-2\tau}$ . . . . .                   | $1.884 \pm 0.014$     | $1.851 \pm 0.018$         | $1.904 \pm 0.024$               | $1.884 \pm 0.012$            | $1.883 \pm 0.011$                   | $1.881 \pm 0.010$                       |
| Age [Gyr] . . . . .                               | $13.830 \pm 0.037$    | $13.761 \pm 0.038$        | $13.64^{+0.16}_{-0.14}$         | $13.800 \pm 0.024$           | $13.797 \pm 0.023$                  | $13.787 \pm 0.020$                      |
| $z_*$ . . . . .                                   | $1090.30 \pm 0.41$    | $1089.57 \pm 0.42$        | $1087.8^{+1.6}_{-1.7}$          | $1089.95 \pm 0.27$           | $1089.92 \pm 0.25$                  | $1089.80 \pm 0.21$                      |
| $r_*$ [Mpc] . . . . .                             | $144.46 \pm 0.48$     | $144.95 \pm 0.48$         | $144.29 \pm 0.64$               | $144.39 \pm 0.30$            | $144.43 \pm 0.26$                   | $144.57 \pm 0.22$                       |
| $100\theta_*$ . . . . .                           | $1.04097 \pm 0.00046$ | $1.04156 \pm 0.00049$     | $1.04001 \pm 0.00086$           | $1.04109 \pm 0.00030$        | $1.04110 \pm 0.00031$               | $1.04119 \pm 0.00029$                   |
| $z_{\text{drag}}$ . . . . .                       | $1059.39 \pm 0.46$    | $1060.03 \pm 0.54$        | $1063.2 \pm 2.4$                | $1059.93 \pm 0.30$           | $1059.94 \pm 0.30$                  | $1060.01 \pm 0.29$                      |
| $r_{\text{drag}}$ [Mpc] . . . . .                 | $147.21 \pm 0.48$     | $147.59 \pm 0.49$         | $146.46 \pm 0.70$               | $147.05 \pm 0.30$            | $147.09 \pm 0.26$                   | $147.21 \pm 0.23$                       |
| $k_D$ [Mpc <sup>-1</sup> ] . . . . .              | $0.14054 \pm 0.00052$ | $0.14043 \pm 0.00057$     | $0.1426 \pm 0.0012$             | $0.14090 \pm 0.00032$        | $0.14087 \pm 0.00030$               | $0.14078 \pm 0.00028$                   |
| $z_{\text{eq}}$ . . . . .                         | $3411 \pm 48$         | $3349 \pm 46$             | $3340^{+81}_{-92}$              | $3407 \pm 31$                | $3402 \pm 26$                       | $3387 \pm 21$                           |
| $k_{\text{eq}}$ [Mpc <sup>-1</sup> ] . . . . .    | $0.01041 \pm 0.00014$ | $0.01022 \pm 0.00014$     | $0.01019^{+0.00025}_{-0.00028}$ | $0.010398 \pm 0.000094$      | $0.010384 \pm 0.000081$             | $0.010339 \pm 0.000063$                 |
| $100\theta_{s,\text{eq}}$ . . . . .               | $0.4483 \pm 0.0046$   | $0.4547 \pm 0.0045$       | $0.4562 \pm 0.0092$             | $0.4490 \pm 0.0030$          | $0.4494 \pm 0.0026$                 | $0.4509 \pm 0.0020$                     |

# Large Scale Structure (DES)

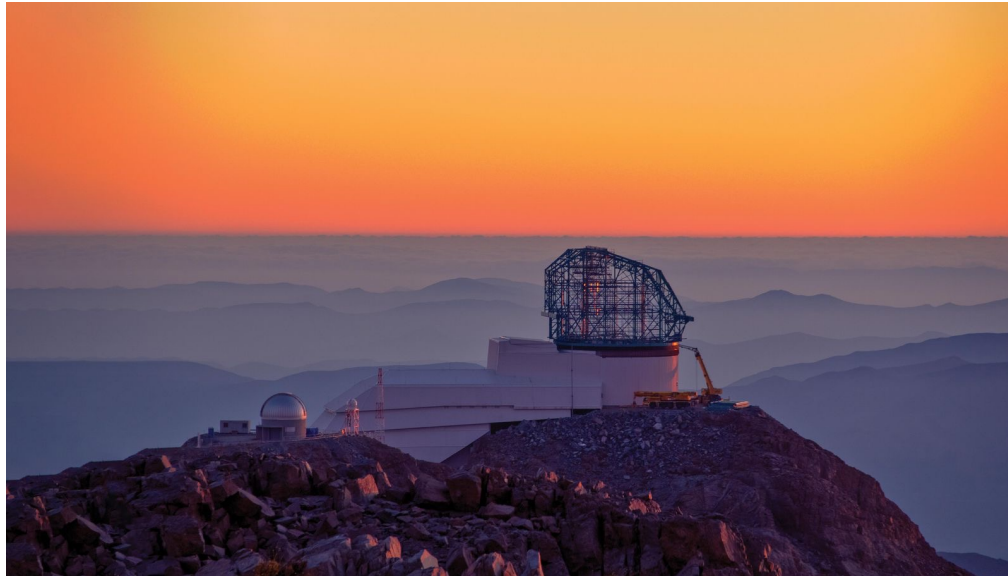




# But we can measure more ...

- Neutrino masses
- Gravitational parameters
- Coupling constants
- Magnetic fields
- Dark matter properties
- Fifth Forces
- Halo properties
- Bias
- Star Formation
- Feedback Mechanism
- ...

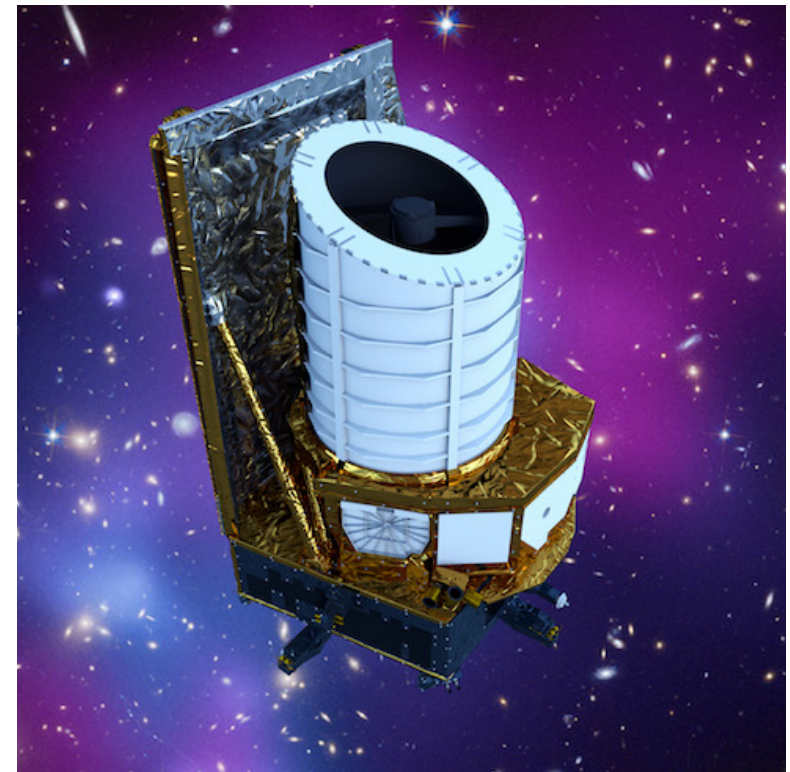
# The new observatories



Vera Rubin Observatory

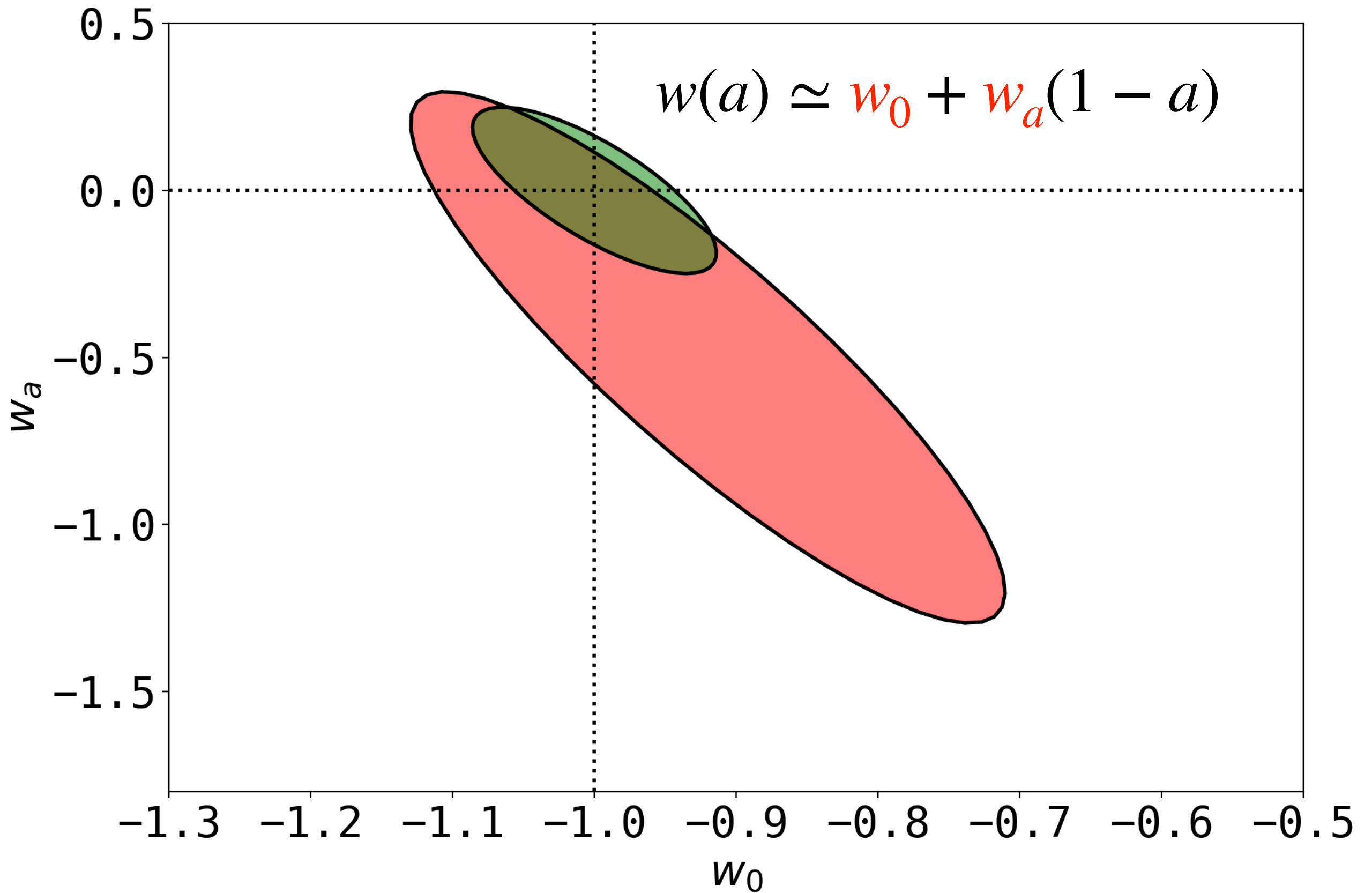


Simons Observatory

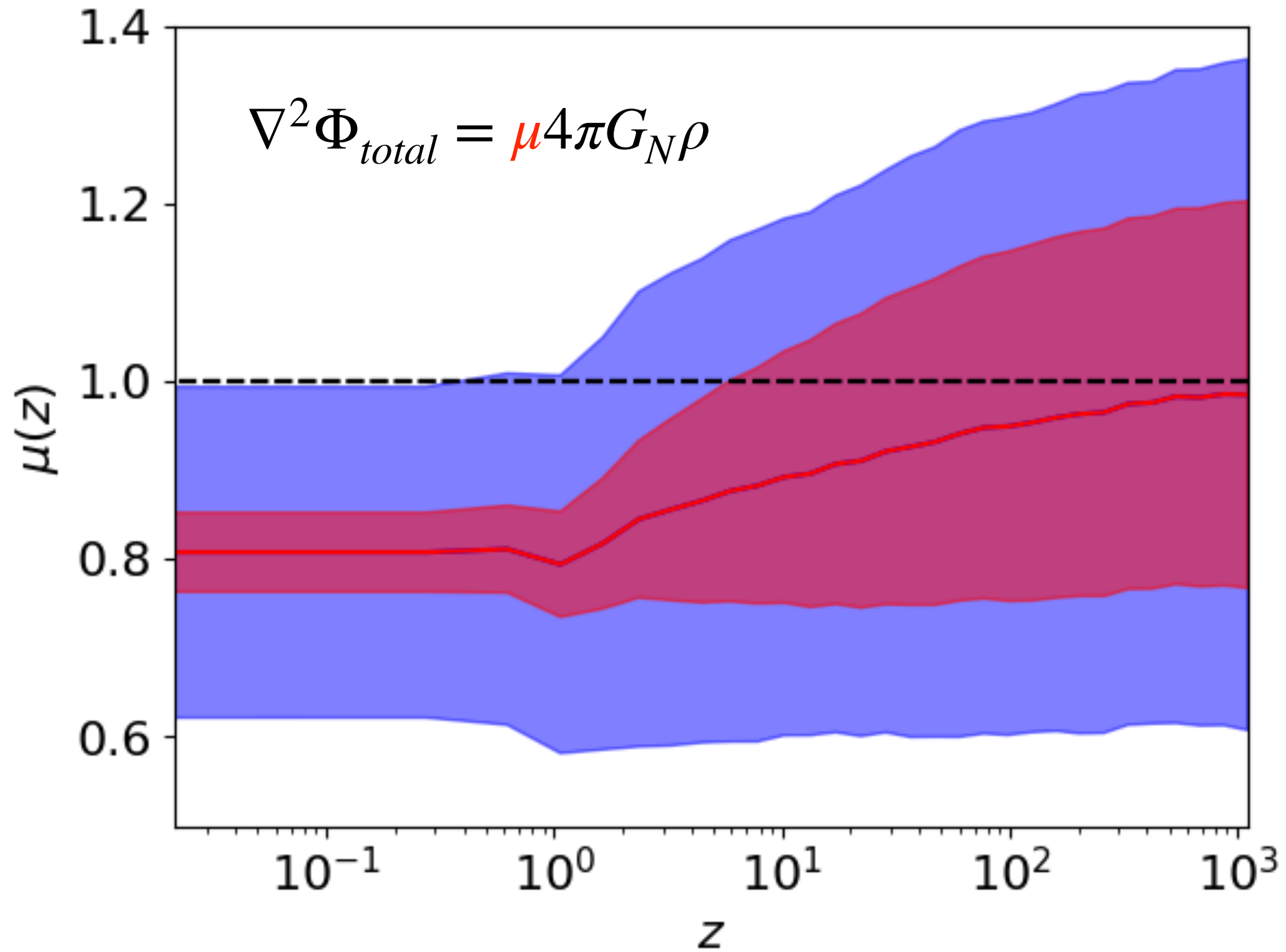


Euclid

Now + 10 years

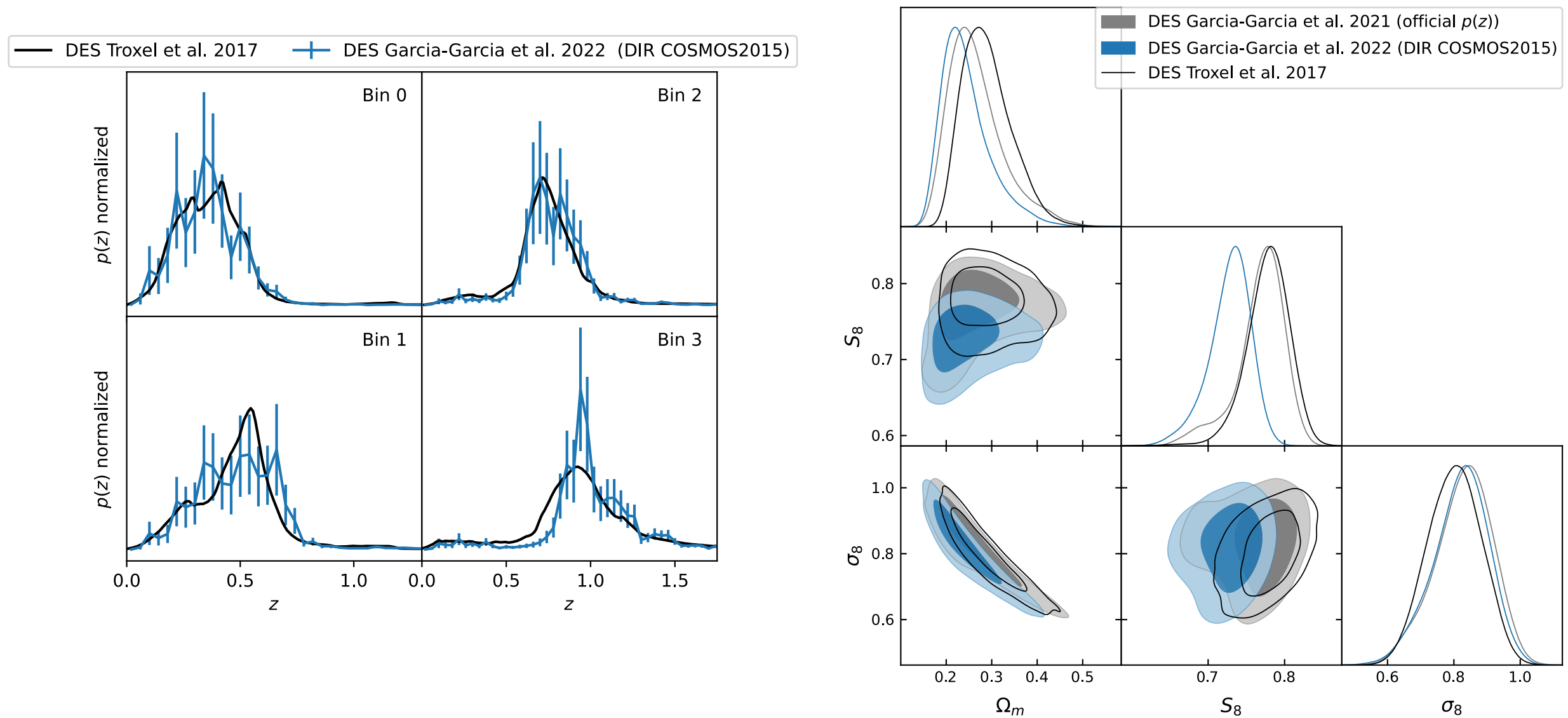


Now + 10 years

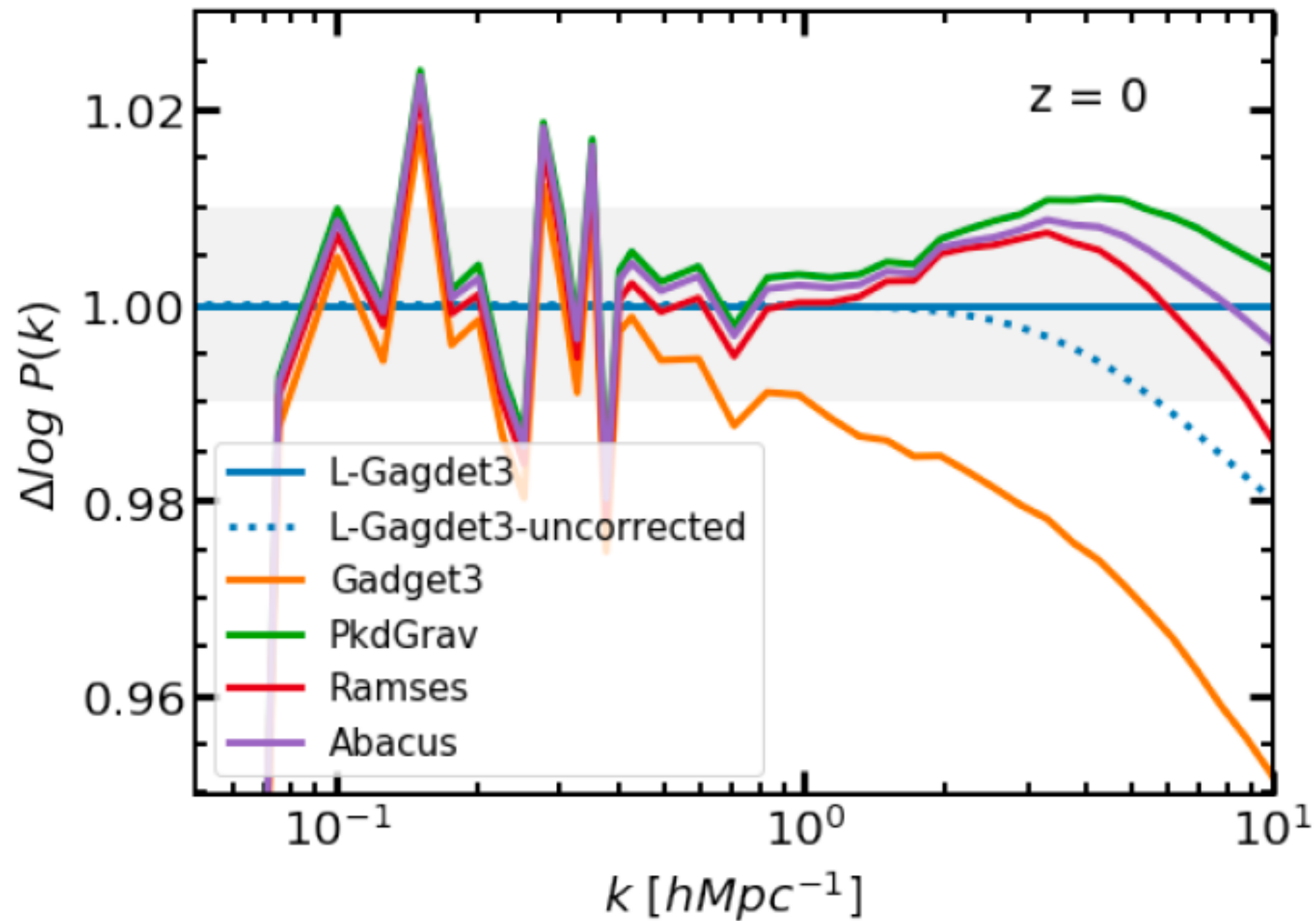


Must Do Better!

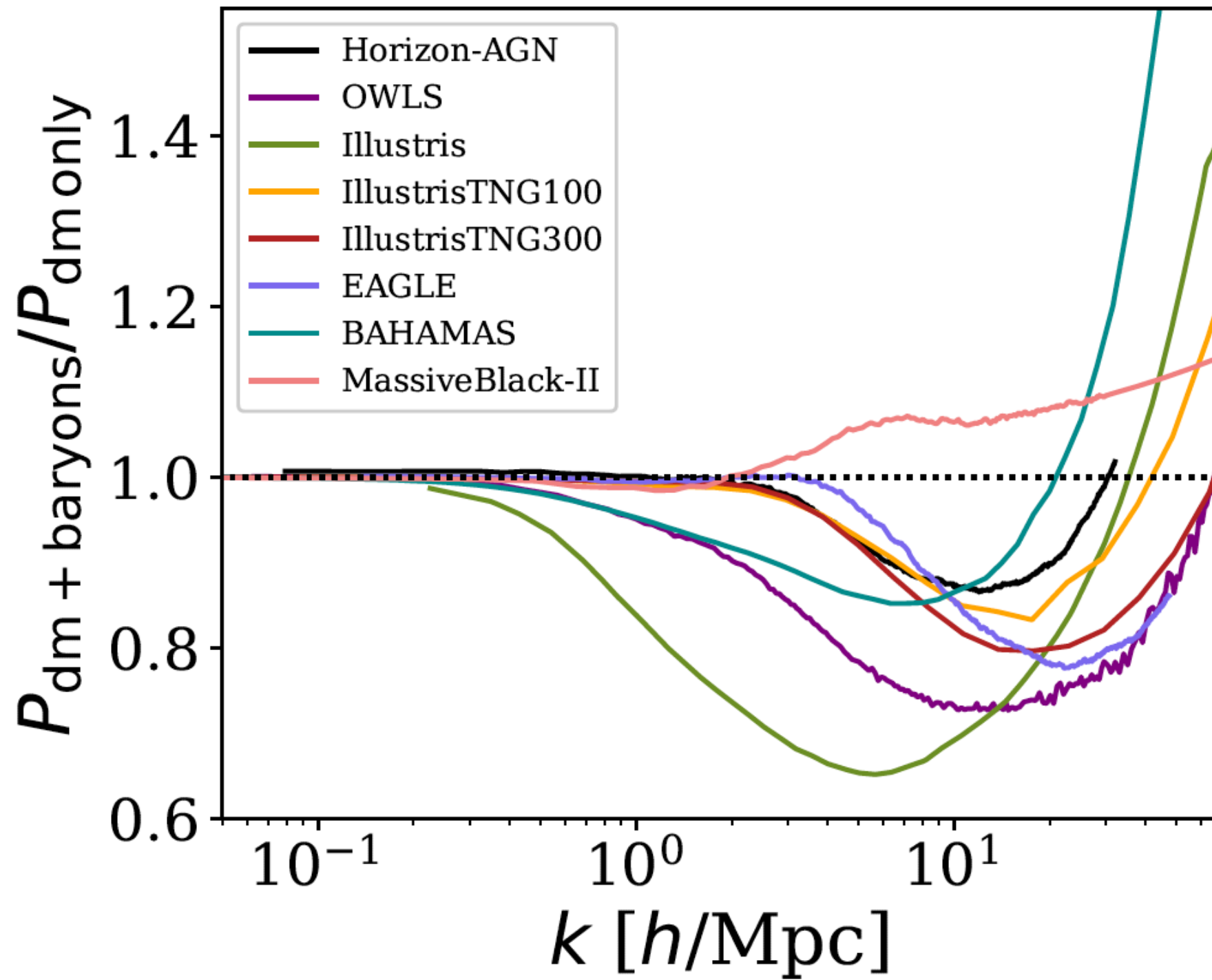
# Systematics...



# Non-linearities...



# Baryons...



We need models/methods which are

- Comprehensive (include all effects)
- Precise
- Accurate
- Fast
- etc...

# Methodological Revolution?

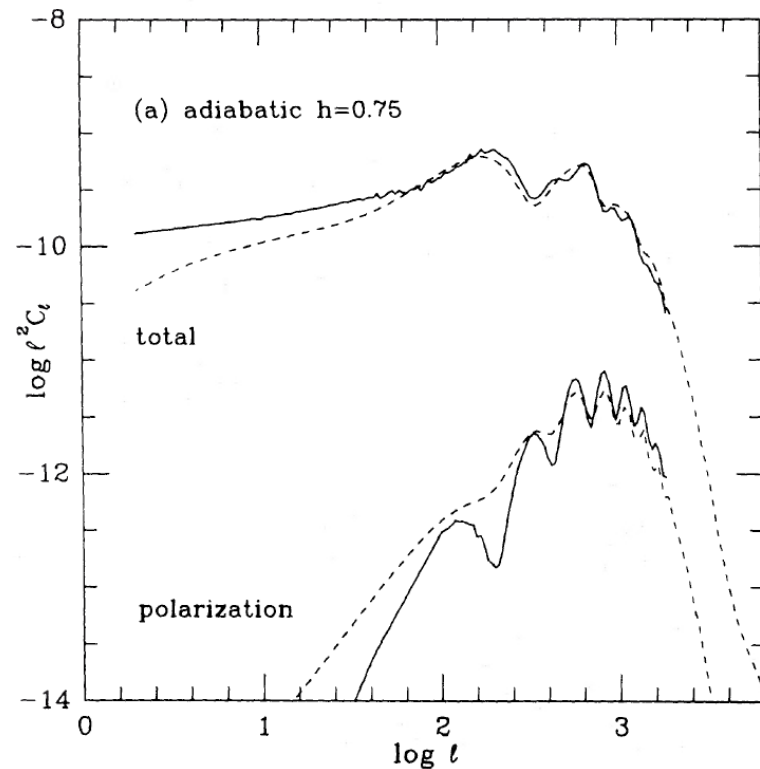
Now: The rise of machine learning ...

- Neural Networks
- Gaussian Processes
- Reinforcement Learning
- Generative Adversarial Networks
- etc...

... and advanced numerical/statistical methods.

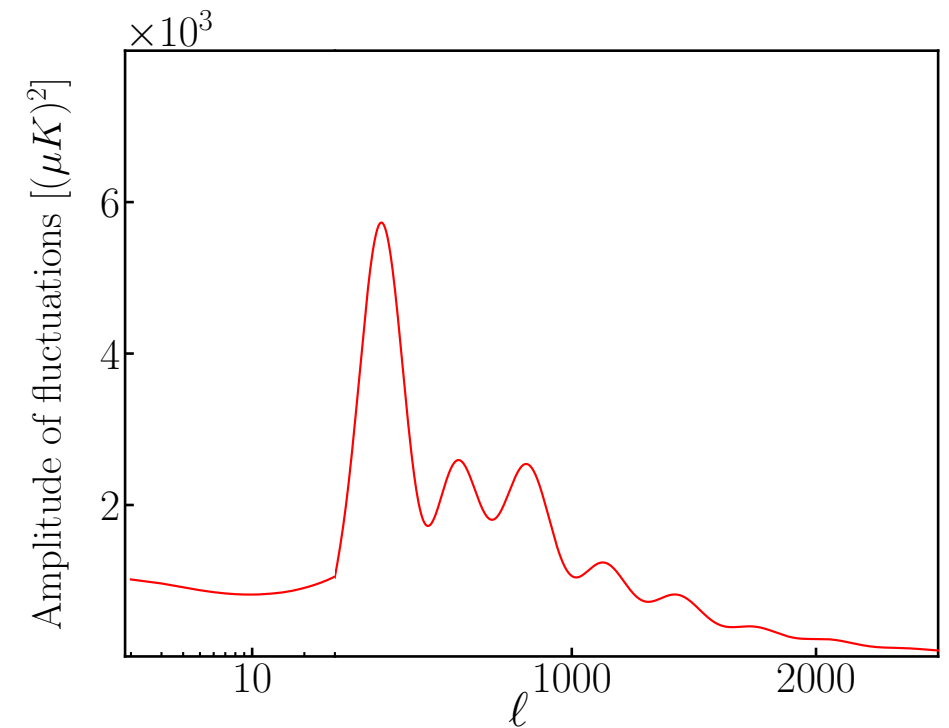
- fast PDE solvers
- GPUs
- Differentiable methods
- Samplers
- Higher order stats
- etc...

# Models: the linear Universe



Bond & Efstathiou 1987

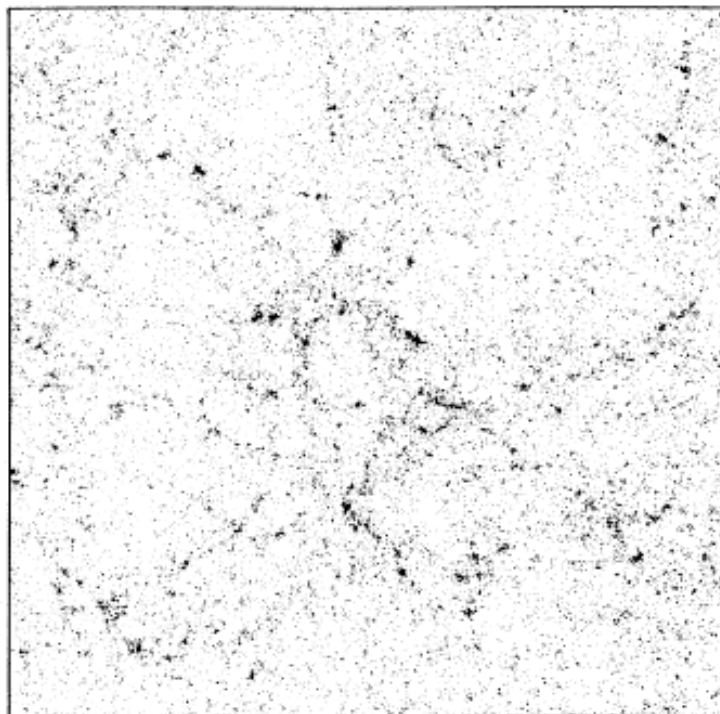
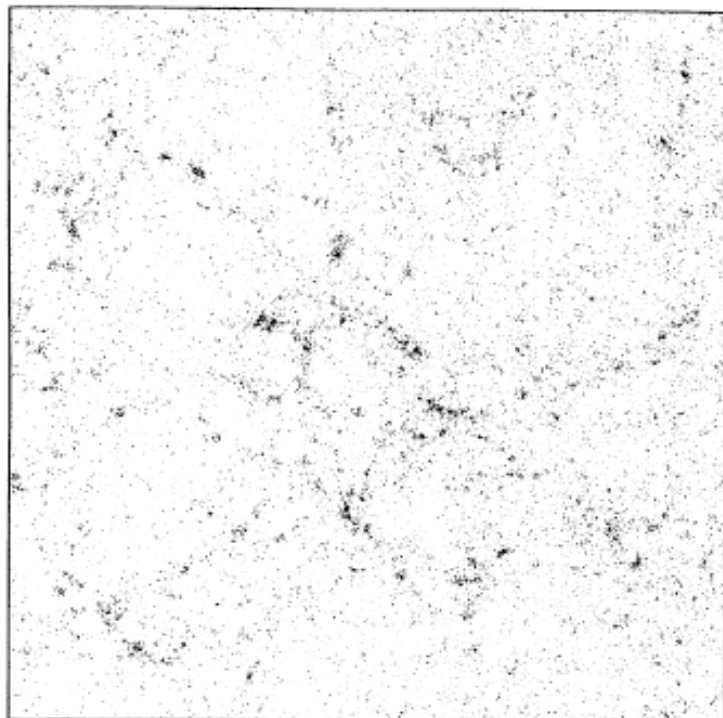
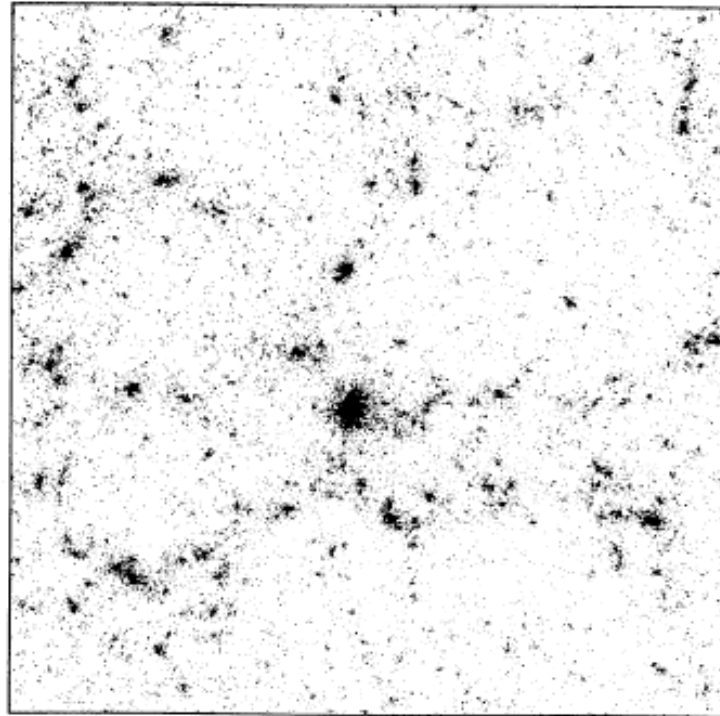
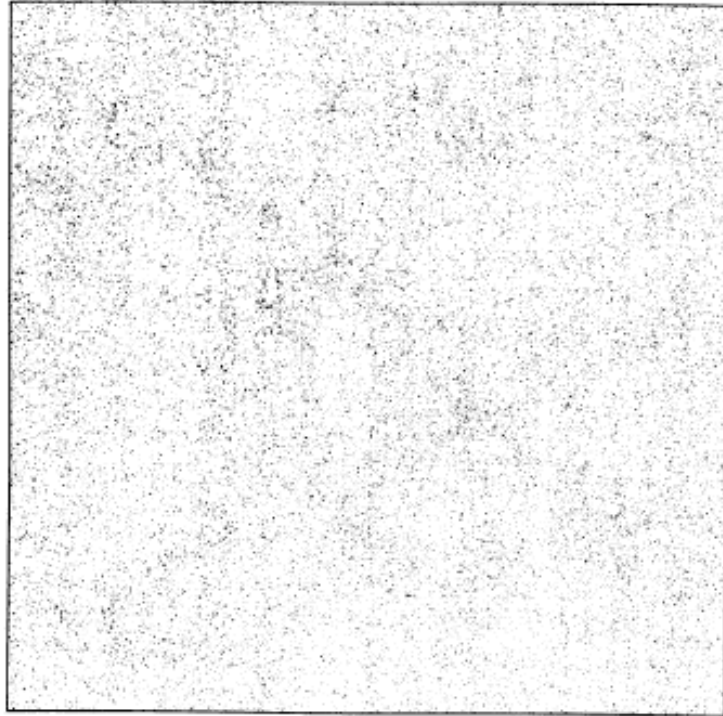
10 hours on a microvax in 1987



seconds on a laptop

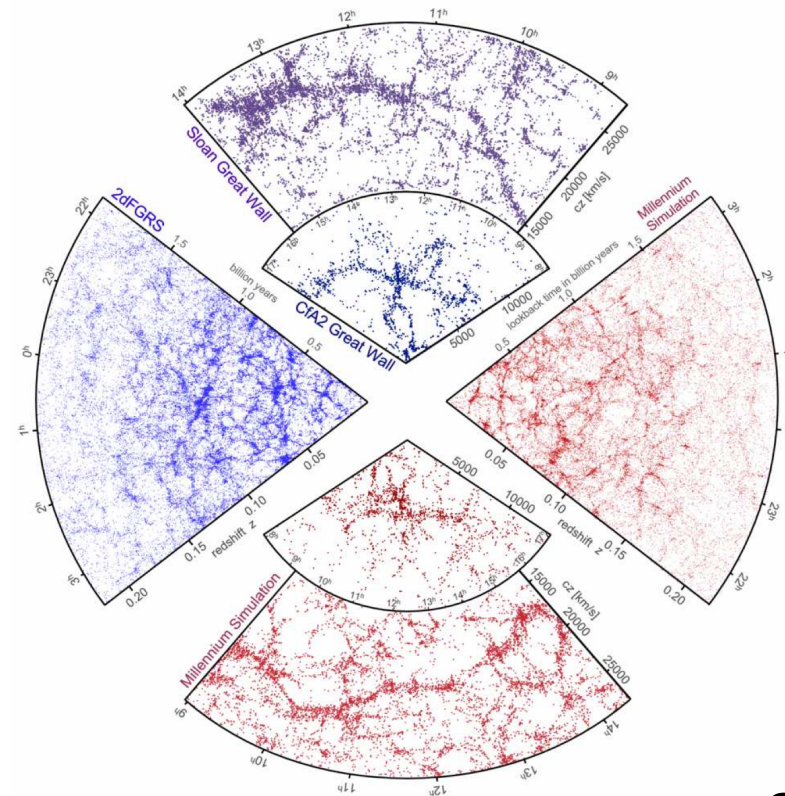
- CLASS
- CAMB
- EFTCAMB
- HiCLASS

# Models: the non-linear Universe



# Simulating the Universe

- GreeM 0910.0121
- HACC 1411.3396
- 2HOT 1310.4502
- Gadget 1203.3216
- Ramses astro-ph/0111367
- ENZO 1307.2265
- Arepo 1503.00562
- PKDGRAV3 1609.08621



Springel et al 2006

Leading to ...

- Aemulus
- Millenium XXL
- Illustrious TNG
- Horizon
- OWLS and Cosmo-OWLS

- CAMELS
- QUIJOTE
- BAHAMAS
- EAGLE
- Tian- $\nu$
- FLAMINGO

# Analytical Models

- EFT of LSS 2110.07539
- LPT bias 2012.04636
- HEFT 1910.07097
- HZPT 2104.10676
- EHM 2004.09515
- Baryonification 1510.06034
- SP(k) 2305.09710
- Kinetic Field Theory 1905.01179

# Emulators or Surrogates

60

BARDEEN *ET AL.*

Vol. 304

Case 1: Cold dark matter, adiabatic fluctuations:

$$T_{\text{CDM, ad, } x}(k) = \frac{\ln(1 + 2.34q)}{2.34q} [1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{-1/4},$$

$$q \equiv \frac{k\theta^{1/2}}{(\Omega_x h^2 \text{ Mpc}^{-1})}. \quad (\text{G3})$$

BBKS 1986

The transfer function is written as a sum of the baryon and cold dark matter contributions at the drag epoch

$$T(k) = \frac{\Omega_b}{\Omega_0} T_b(k) + \frac{\Omega_c}{\Omega_0} T_c(k). \quad (8)$$

The CDM transfer function can be solved exactly in terms of hypergeometric functions that are more conveniently approximated by the following form:

$$T_c \rightarrow \alpha_c \frac{\ln 1.8\beta_c q}{14.2q^2}, \quad (9)$$

$$q = \left( \frac{k}{\text{Mpc}^{-1}} \right) \Theta_{2.7}^2 (\Omega_0 h^2)^{-1} = \frac{k}{13.41 k_{\text{eq}}}, \quad (10)$$

where  $\alpha_c$  and  $\beta_c$  are fitted by

$$\alpha_c = a_1^{-\Omega_b/\Omega_0} a_2^{-(\Omega_b/\Omega_0)^3},$$

$$a_1 = (46.9\Omega_0 h^2)^{0.670} [1 + (32.1\Omega_0 h^2)^{-0.532}],$$

$$a_2 = (12.0\Omega_0 h^2)^{0.424} [1 + (45.0\Omega_0 h^2)^{-0.582}], \quad (11)$$

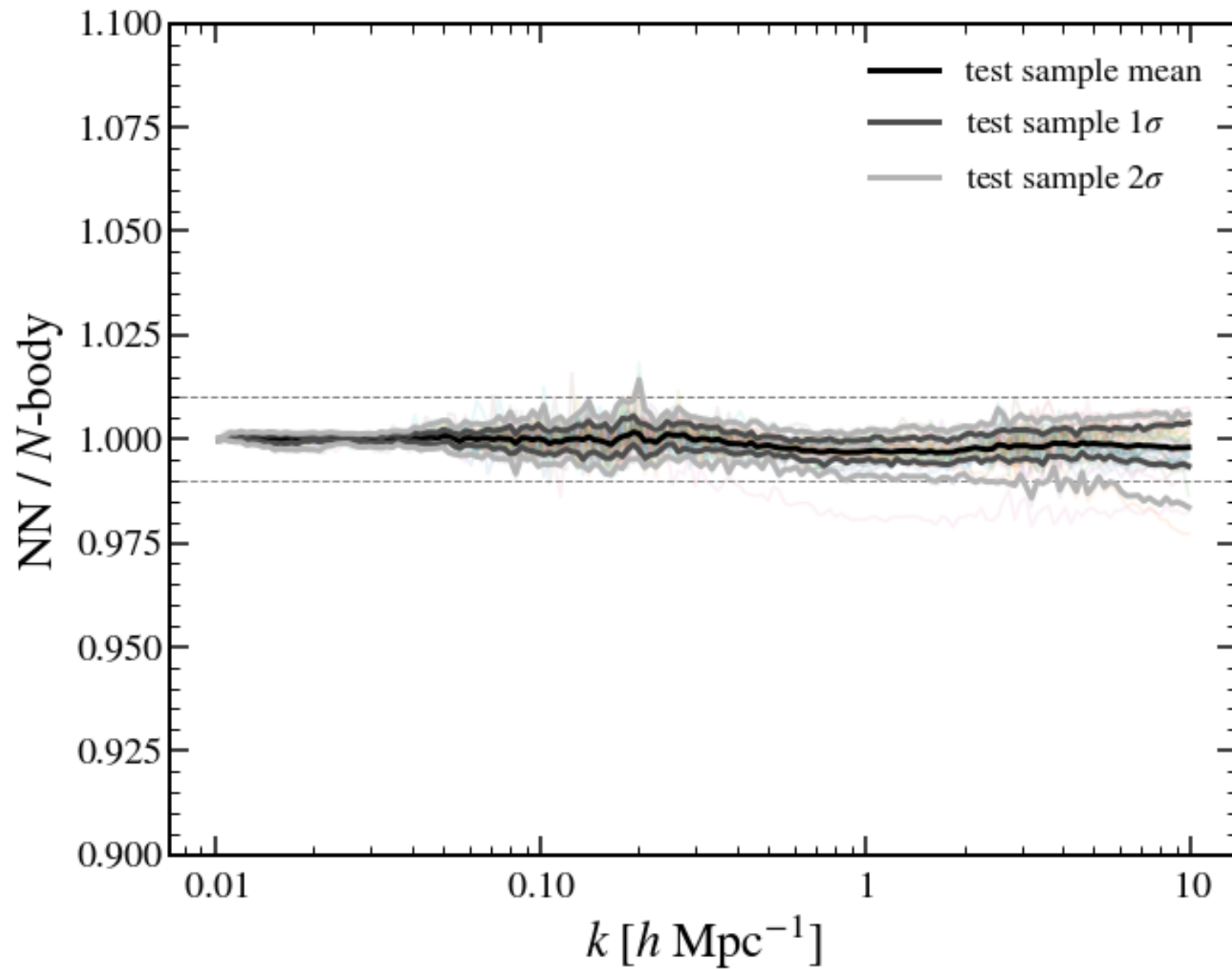
$$\beta_c^{-1} = 1 + b_1 [(\Omega_c/\Omega_0)^{b_2} - 1],$$

$$b_1 = 0.944 [1 + (458\Omega_0 h^2)^{-0.708}]^{-1},$$

$$b_2 = (0.395\Omega_0 h^2)^{-0.0266}. \quad (12)$$

Eisenstein and Hu 1998

# Emulators or Surrogates



Zennaro et al 2023

# An explosion of Emulators or Surrogates

- BACCOEmulator (NN) 2004.06245
- EuclidEmu (PCE) 2010.11288
- PKEmu (GP) 2105.02256
- CosmoPower (NN) 2106.03846
- CAPSE & EFFORT (julia, NN) 2206.14208
- MiraTitan (GP) 1508.02654
- Aemulus (GP and PCE) 1804.05867
- COMET (GP) 2208.01070
- BCEmu (NN) 2108.08863
- MFEmu (GP) 2306.03144

GP - Gaussian Processes

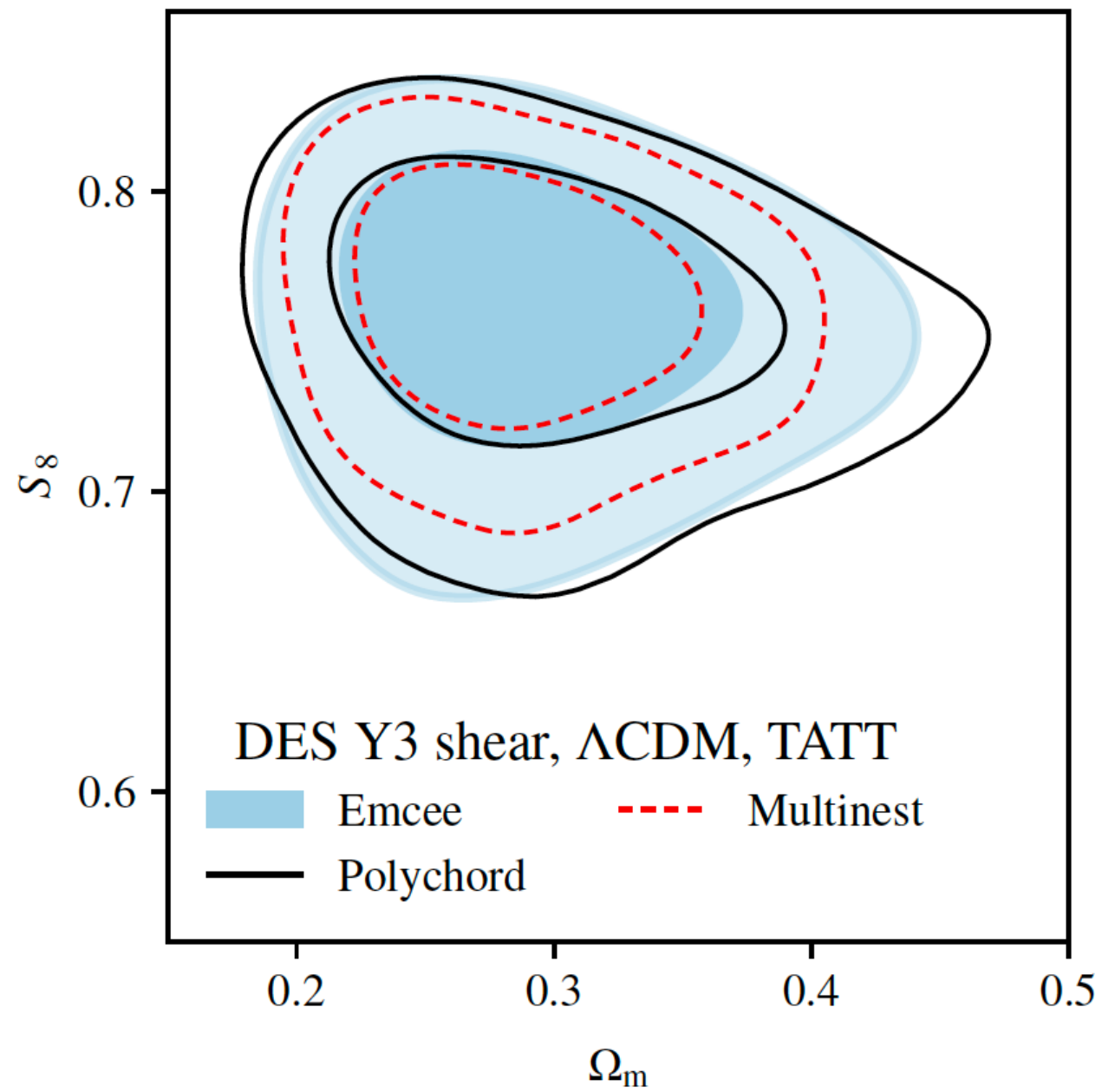
NN - Neural Nets

PCE - Polynomial Chaos Expansion

# Samplers

- CosmoMC astro-ph/0205436
- MontePython 1804.07261
- emcee 1202.3665
- Cosmosis 1409.3409
- Cobaya 2005.0529
- Polychord 1506.0017
- CosmoNest astro-ph/0608184
- MultiNest 0809.3437
- CosmoHammer 1206516
- NUTS 1111.4246
- HMC 1701.0243
- MCHMC 2212.08549
- Langevin-HMC 2205.14240
- Black-BOX VI 1601.00670
- VI 1603.00788
- ADVI 2108.03782
- MUSE 2112.09354

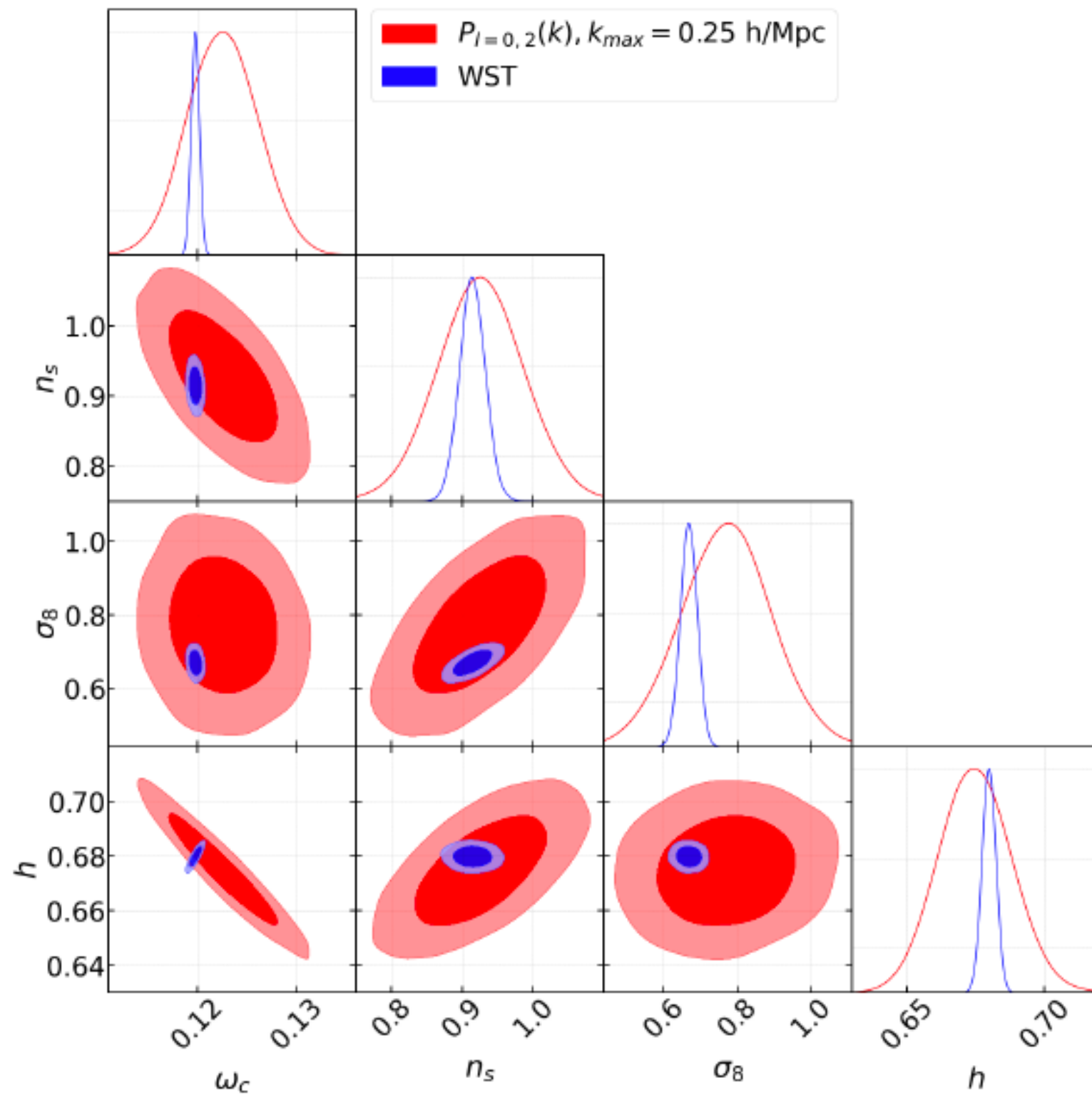
# Samplers ...



# Other Statistics

- Bispectrum
- Trispectrum
- Peaks
- Edgeworth Expansion
- Cumulants
- Minkowski functionals
- Wavelet Transform Coefficients
- etc

# Other Statistics



# Non Gaussian Optimism!

## Cosmological Collider Physics

Nima Arkani-Hamed and Juan Maldacena

Institute for Advanced Study, Princeton, NJ 08540, USA

Citations per year



⇒ 478 citations

# Optimism tempered with pragmatism...

Example: target  $f_{NL} \sim \mathcal{O}(1)$

History:

CMB

but...

$$\left\{ \begin{array}{l} \bullet \text{ MAXIMA: } \sigma(f_{NL}) \sim 10^3 \text{ astro-ph/0107588} \\ \bullet \text{ WMAP: } \sigma(f_{NL}) \sim 10^2 \text{ 1005.5568} \\ \bullet \text{ Planck: } \sigma(f_{NL}) \sim 10^1 \text{ 1905.0569} \end{array} \right.$$

LSS

$$\left\{ \begin{array}{l} \bullet \text{ WMAP: } \sigma(f_{NL}) \sim 10^2 \text{ 1005.5568} \\ \bullet \text{ BOSS: } \sigma(f_{NL}) \sim 10^2 \text{ 1905.0569} \end{array} \right.$$

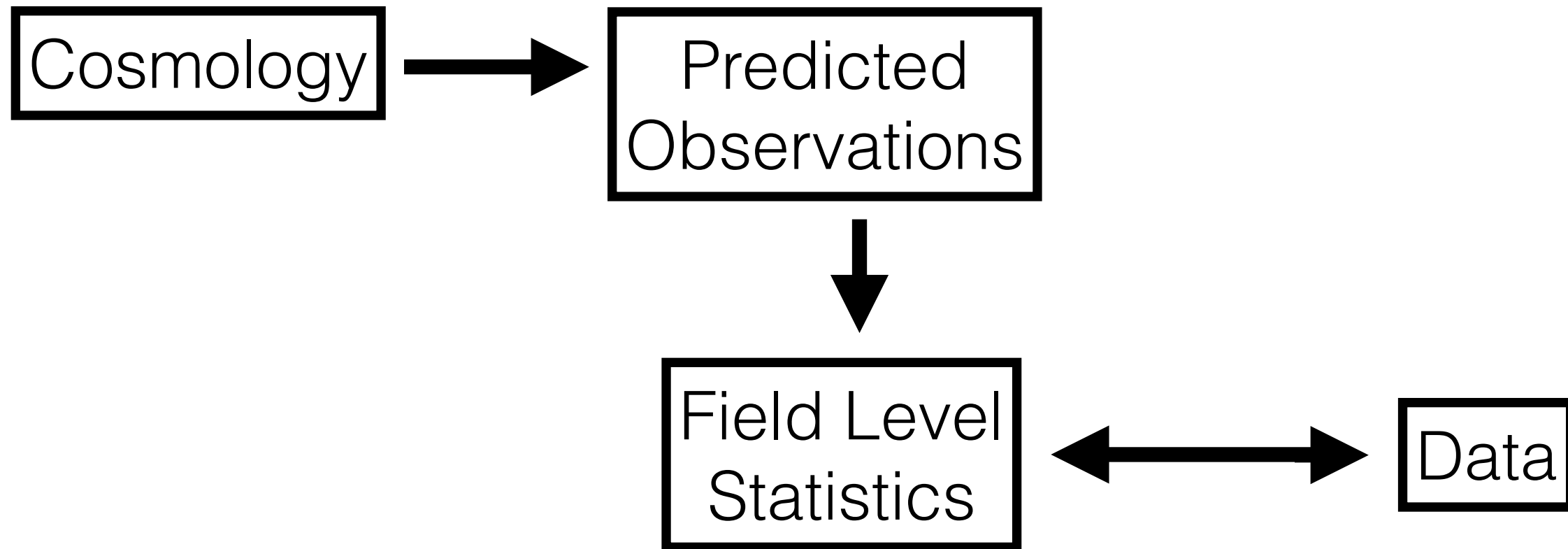
# Differentiable Methods

- JAXCOSMO 2302.05163
- CosmoPower-JAX 2305.06347
- Limberjack.jl
- CMBLensing.jl
- Bolt.jl

Using

- Turing (in julia)
- Stan (in C)
- NumPyro (in JAX)

# Bayesian forward modelling and Field Level Inference



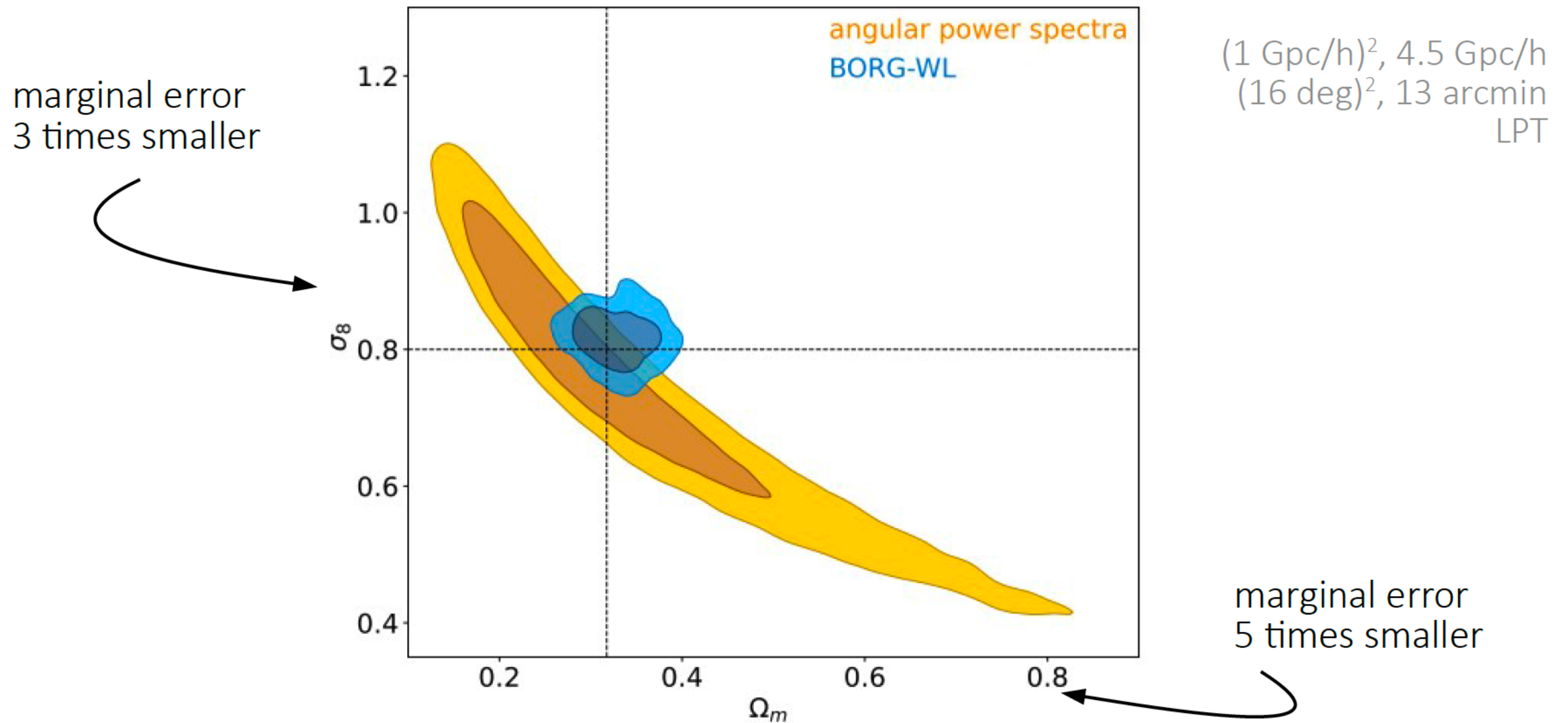
Statistics pixel by pixel can access all information.

Probabilistic forward model ➡ Simulations constrained by data

# Field Level Inference

- Aquila 0911.2496
- Elucid 1407.3451
- Almanac 2210.13260
- LEFTfield 2303.09876
- CCA 2302.14101
- KARMMMA 2105.14699
- DLPosterior 2201.05561
- FGPA 2305.10428
- FORECAST 2305.19166

# Field Level Inference

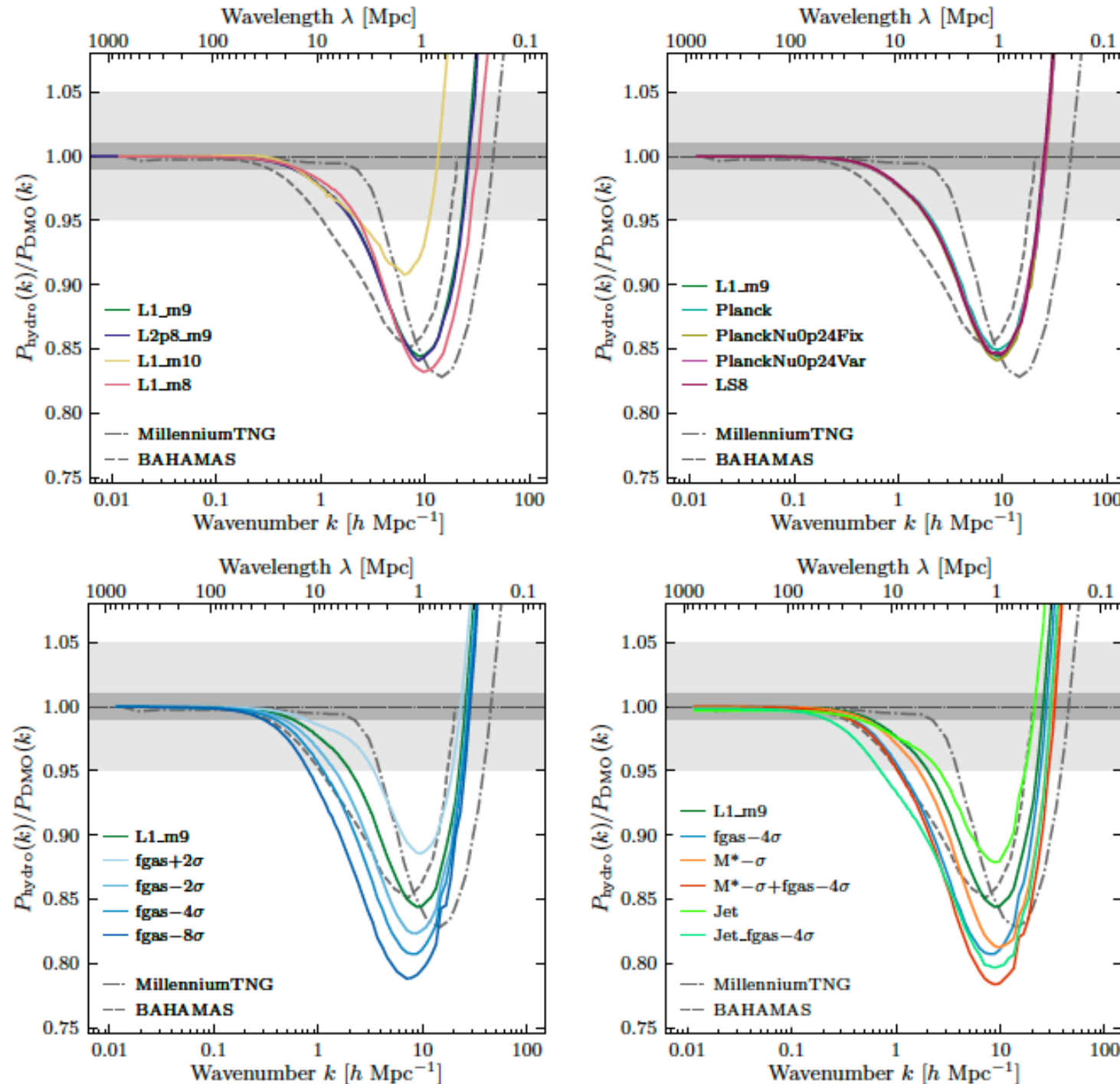


Field-level approach **lifts degeneracy** by extracting more information from the data

Porqueres+2023

# Questions for the future

Is there a limit to how deep into the non-linear/baryonic regime we can go? Should we cut our losses?



# Questions for the future

Is there a limit to how deep into the non-linear/baryonic regime we can go? Should we cut our losses?

How important are analytic methods versus emulators?

Should we all aim for differentiable methods?

Can we already see a clear winner for ML methods?

How important are field level methods versus summary statistics?

Should we start using new languages (Julia) or architectures?

*Special Section:* HOW TO HAVE FUN OUTDOORS

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