UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

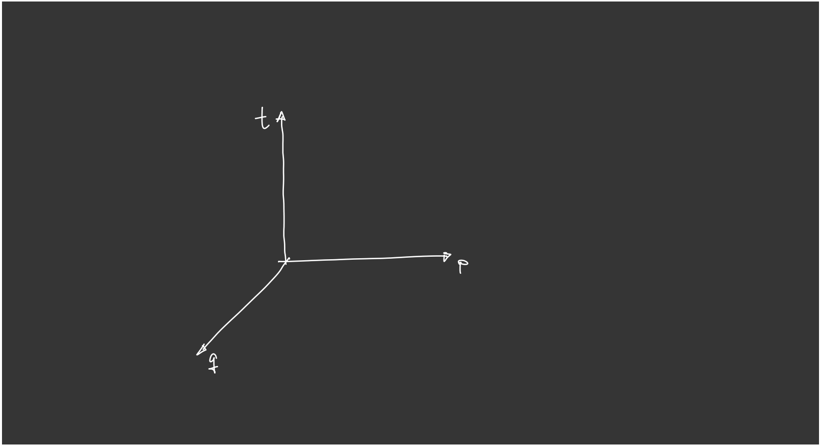
Kinetic Field Theory For Cosmic Structure Formation

Matthias Bartelmann

Institute for Theoretical Physics, Heidelberg University

Perspectives on Large-Scale Structure
CEICO, Prague, June 2023

Kinetic Field Theory (1)



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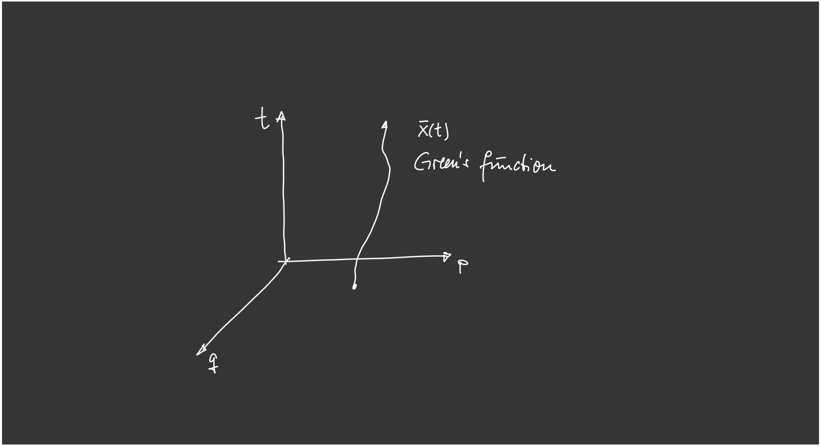

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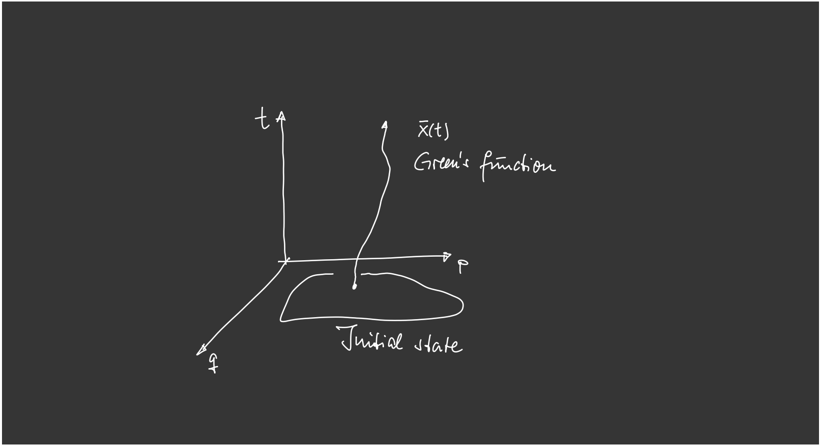

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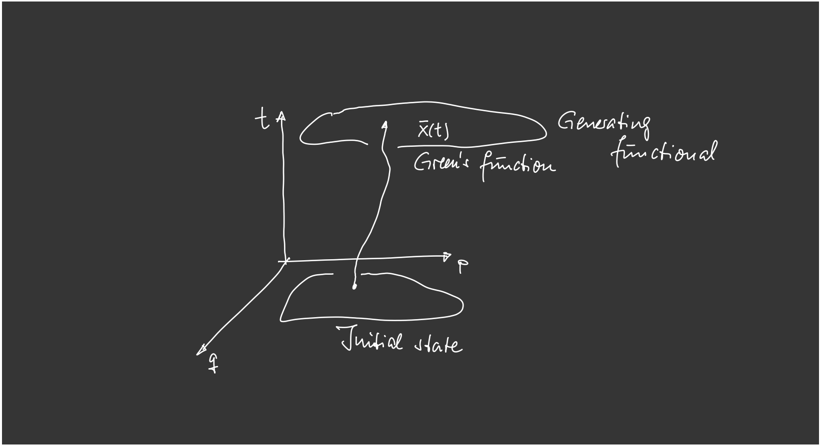

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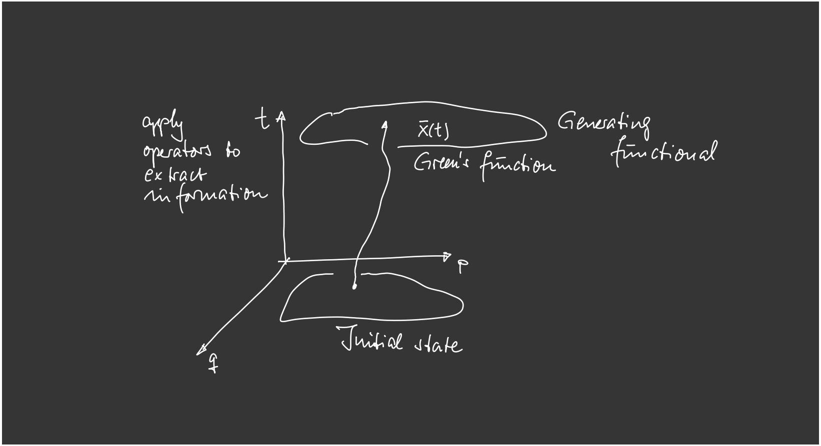

Kinetic Field Theory (2)

Generating functional:

$$Z[J] = \int P(x,t)\, e^{i(x,J)}$$

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Generating functional:

$$Z[J] = \int P(x,t)\, e^{i(x,J)}$$

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Hamilton

Kinetic Field Theory (2)

Generating functional:

$$Z[J] = \int P(x,t)\, e^{i(x,J)}$$

$$= \int\int \underbrace{P(x,t\,|\,x^{(i)})}_{\delta_D(x-\bar{x}(x^{(i)}))} \underbrace{P(x^{(i)})\, dx^{(i)}}_{d\Gamma}\, e^{i(x,J)}$$

$$= \int d\Gamma\, e^{i(\bar{x},J)}$$

Kinetic Field Theory (3)

Trajectories:

$$\bar{x}(t) = \underbrace{G(t,0)\,x^{(i)}}_{\text{free},\ \bar{x}_0} + \underbrace{\int_0^t G(t,t')\,F(t')\,dt'}_{\text{interacting},\ \bar{x}_I}$$

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Operator

$$= e^{i\hat{S}_I} \underbrace{\int d\Gamma\, e^{i(\mathcal{J},\bar{x}_0)}}_{\text{free}, Z_0} = e^{i\hat{S}_I} Z_0[\mathcal{J}]$$

Kinetic Field Theory (4)

Interactions:

$$e^{i\hat{S}_I} Z_0[J] \longrightarrow \sum_{n=0}^{\infty} \frac{(i\hat{S}_I)^n}{n!} Z_0[J]$$

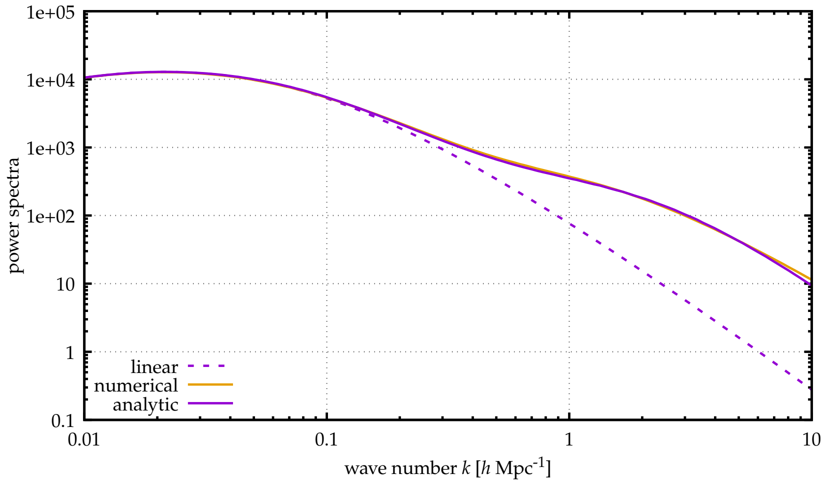
perturbation theory

Kinetic Field Theory (4)

Interactions:

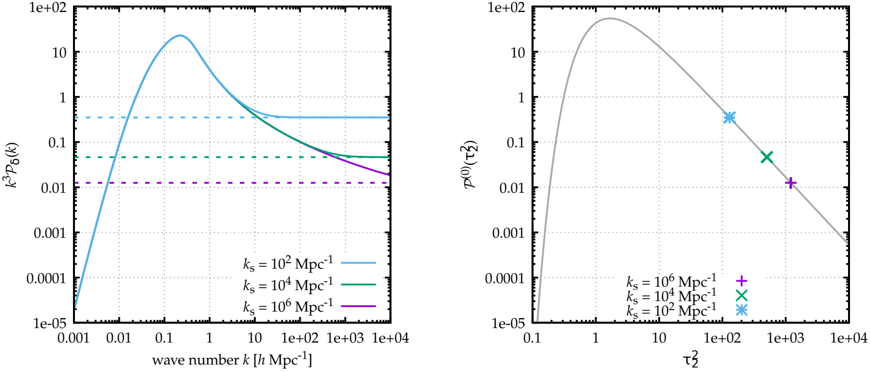
$$
\begin{tikzcd}
 & \displaystyle\sum_{n=0}^{\infty} \frac{(i\hat{S}_I)^n}{n!}\, Z_0[J] & \text{perturbation theory} \\
e^{i\hat{S}_I}\, Z_0[J] \arrow[ur] \arrow[dr] & & \\
 & e^{i\langle \hat{S}_I \rangle}\, Z_0[J] & \text{mean field}
\end{tikzcd}
$$

Mean-Field Approximation



Konrad, S. & MB 2022; MB et al. 2021

Asymptotic Behaviour



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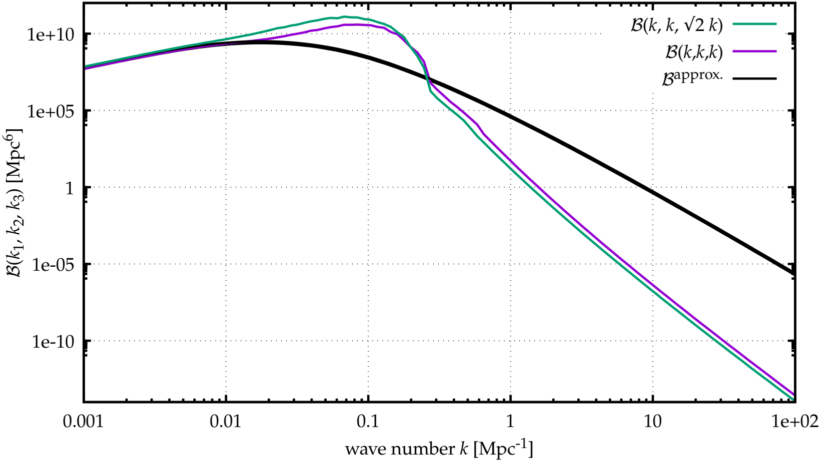
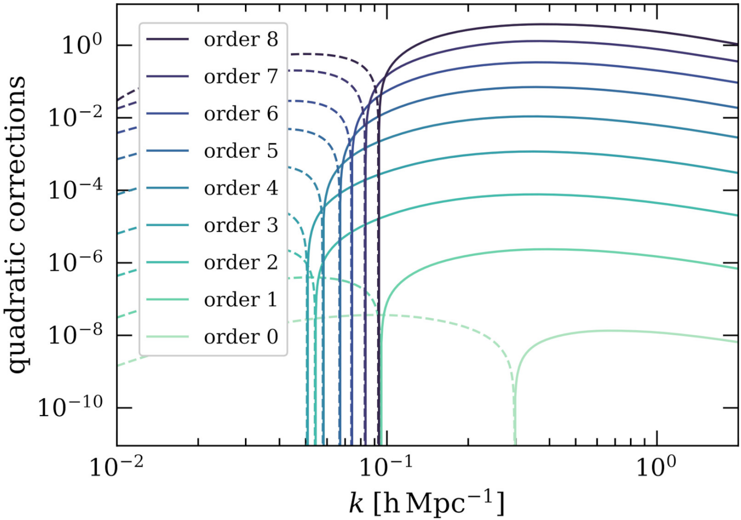


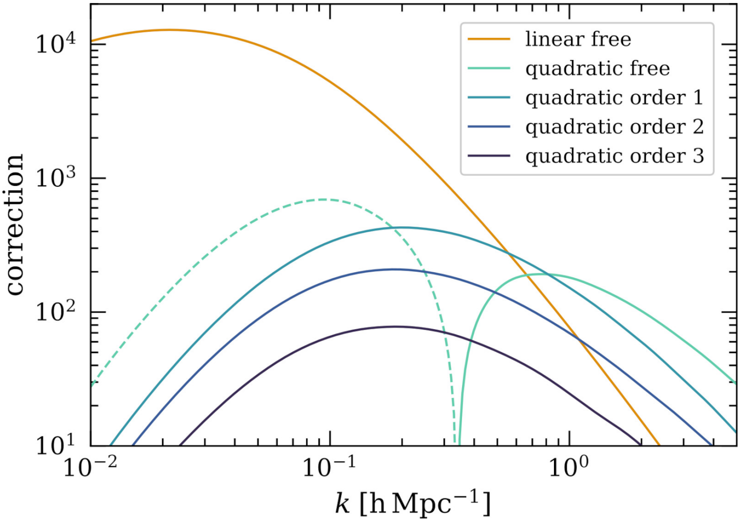
Waibel, R., MSc thesis, unpublished

Perturbation Theory



Pixius, C., PhD thesis; Pixius, C. et al. 2022

Perturbation Theory



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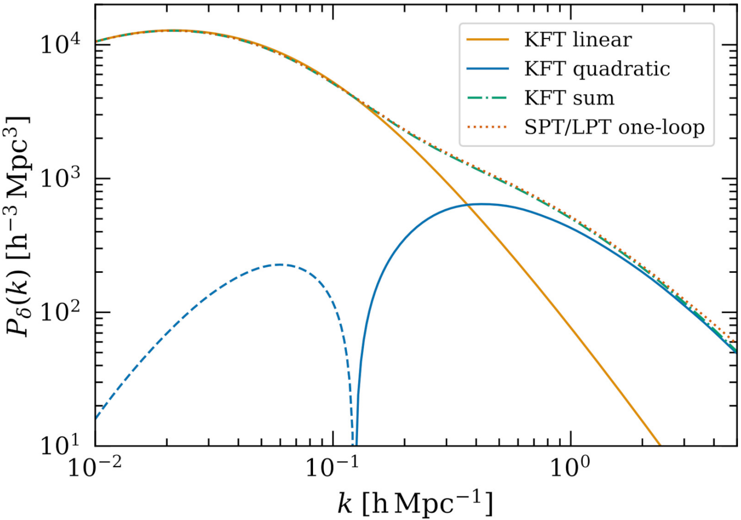

Pixius, C., PhD thesis

Summary

- KFT: new statistical approach to classical, non-equilibrium systems
- avoids shell-crossing problem by construction
- mean-field approach successful in recovering non-linear power spectrum
- rigorous statements on asymptotic behaviour of cosmic structures
- perturbation theory can be driven to high orders
- no free parameters
- generalization to different cosmologies, dark-matter models, gravity theories easily possible ($\rightarrow$ Elena Kozlikin's talk)